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# Central Bank Digital Currency and Financial Inclusion

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**Central Bank Digital Currency and Financial Inclusion**
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**ABSTRACT:** In this paper, we develop a model incorporating the impact of financial inclusion to study the implications of introducing a retail central bank digital currency (CBDC). CBDCs in developing countries (unlike in advanced countries) have the potential to bank large unbanked populations and boost financial inclusion which can increase overall lending and reduce bank disintermediation risks. Our model captures two key channels. First, CBDC issuance can increase bank deposits from the previously unbanked by incentivizing the opening of bank accounts for access to CBDC wallets (offsetting potential flows from deposits to CBDCs among those already banked). Second, data from CBDC usage allows for the building of credit to reduce credit-risk information asymmetry in lending. We find that CBDC can increase overall lending if (1) bank deposit liquidity risk is low, (2) the size and relative wealth of the previously unbanked population is large, and (3) CBDC is valuable to households as a means of payment or for credit-building. CBDC can still be optimal for household welfare even when overall lending decreases as households benefit from the value of using CBDC for payments, CBDC provides an alternative "safe" savings vehicle, and CBDC generates greater surplus in lending by reducing credit-risk information asymmetry. Most countries are considering a "two-tier" CBDC model, where central banks issue CBDC to commercial banks which in turn distribute them to consumers. If non-bank payment system providers can distribute CBDC, fewer funds will flow into deposit accounts from the unbanked because a bank account is no longer needed to access CBDC. If CBDC data is shareable with banks, those without bank accounts can still build credit and access lower interest rate loans. This design is optimal for welfare if the gains from greater access to CBDC outweigh the contraction in lending.

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WORKING PAPERS

# Central Bank Digital Currency and Financial Inclusion

Prepared by Brandon Joel Tan

# Central Bank Digital Currency and Financial Inclusion*

Brandon Joel Tan†

March 2023

**Abstract**

Boosting financial inclusion is one of the main motivations for issuing retail central bank digital currencies (CBDCs) among developing economies. In this paper, we develop a model incorporating the impact of financial inclusion to study the implications of introducing CBDC. One of the most frequently raised concerns regarding CBDC issuance is the disintermediation of banks and impact on the overall supply of credit. However, CBDCs in developing countries have the potential to bank large unbanked populations and boost financial inclusion which can increase overall lending and reduce bank disintermediation risks. Our model captures two key channels. First, CBDC issuance can increase bank deposits from the previously unbanked by incentivizing the opening of bank accounts for access to CBDC wallets (offsetting potential flows from deposits to CBDCs among those already banked). Second, data from CBDC usage allows for the building of credit to reduce credit-risk information asymmetry in lending. We find that CBDC can increase overall lending if (1) bank deposit liquidity risk is low, (2) the size and relative wealth of the previously unbanked population is large, and (3) CBDC is valuable to households as a means of payment or for credit-building. CBDC can still be optimal for household welfare even when overall lending decreases as households benefit from the value of using CBDC for payments, CBDC provides an alternative "safe" savings vehicle, and CBDC generates greater surplus in lending by reducing credit-risk information asymmetry. Most countries are considering a "two-tier" CBDC model, where central banks issue CBDC to commercial banks which in turn distribute them to consumers. If non-bank payment system providers can distribute CBDC, fewer funds will flow into deposit accounts from the unbanked because a bank account is no longer needed to access CBDC. If CBDC data is shareable with banks, those without bank accounts can still build credit and access lower interest rate loans. This design is optimal for welfare if the gains from greater access to CBDC outweigh the contraction in lending.

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# 1 Introduction

As the demand for digital forms of payment for retail purposes has grown and the use of cash has declined (Auer and Böhme, 2020), central banks around the world have begun to explore the issuance of central bank digital currencies (CBDCs). 90% of central banks worldwide are exploring the issuance of CBDCs, with 60% conducting experiments or proofs of concept (Auer et al., 2020).

There are a range of motivations for issuing retail CBDCs including maintaining monetary sovereignty, strengthening monetary policy pass-through, combating the illicit use of money, strengthening competition for e-money payment providers, and improving payments efficiency and safety which feature prominently in reports published by central banks in advanced economies (Bank of Denmark, 2017, Riksbank, 2018, Bank of Israel, 2018, Norges Bank, 2019, European Central Bank, 2020, and Bank of England, 2021). Distinct from advanced economies, boosting financial inclusion stands out as one of the main objectives for retail CBDCs among emerging market and low-income economies (Kosse and Mattei, 2022).[1]

The most frequently raised concern in the CBDC discussion is the potential risk of bank disintermediation and contraction in the overall supply of credit if the issuance of CBDC results in bank deposits flowing quickly into CBDC accounts (Carapella and Flemming, 2020; Mancini-Griffoli et al., 2018). As summarized by Infante et al. (2022), the literature research studying the effects of CBDC on the banking sector (Andolfatto, 2021, Keister and Sanches, 2022, Chiu et al., 2022, Whited et al., 2022, Garratt et al., 2022, Piazzesi and Schneider, 2020) has focused on four main factors: the competitiveness of the banking sector, CBDC remuneration, wholesale funding, and CBDC account limits.

Our paper seeks to break new ground in this rapidly growing literature by incorporating financial inclusion as a factor, which is especially important for emerging market and low-income economies and overlooked by existing work. While there are high levels of bank account ownership in many advanced economies, a large share of the population in emerging market and developing economies are unbanked (Demirgüç-Kunt et al., 2022). As a result, CBDCs in emerging market and low-income economies have the potential to bank their unbanked populations and boost financial inclusion which can increase overall lending and reduce bank disintermediation risks.

We develop a model to incorporate the impact of financial inclusion to study the impli-

[1] A BIS survey of 81 central banks found financial inclusion to be a top priority for CBDC development amongst emerging market and developing economies (Kosse and Mattei, 2022).

cations of introducing CBDC. In our model, CBDC is valuable as a means of payment and has zero liquidity risk. We assume a "two-tier" CBDC model (which most countries are considering) where central banks issue CBDC to commercial banks which in turn distribute them to consumers.[2] Households can open a bank account at a fixed cost and a bank account gives them access to a deposit account and a CBDC wallet. Households maximize utility over two periods and can save either in cash, a deposit account, or a CBDC wallet. Households seek a loan from the bank to invest in a production technology and are of two types with a "good" or "bad" credit-risk profile. The use of CBDC for payments allows the bank to distinguish between types.[3] The commercial bank lends to "good" type borrowers at lower interest rates allowing them to take out larger loans.

The model captures two key channels by which financial inclusion matters for the impact of CBDC on household welfare, bank disintermediation, and overall lending. First, CBDC issuance can increase bank deposits from the previously unbanked by incentivizing the opening of bank accounts for access to CBDC wallets. The inflow of new deposits from the unbanked can offset potential flows from deposits to CBDCs among those already banked. Second, data from the use of CBDC for payments can help borrowers establish a credit history reducing credit-risk information asymmetry for lending.[4]

We outline conditions under which CBDC issuance is optimal. We find that CBDC can increase overall lending if (1) bank deposit liquidity risk (disintermediation risk) is low (2) the size and relative wealth of the previously unbanked population is large, and (3) CBDC is valuable to households as a means of payment or for credit building. CBDC can still be optimal for household welfare when overall lending decreases. This is because although households can borrow less for investment in the production technology reducing expected profits, CBDC issuance directly improves welfare through three channels. First, households gain from the value of using CBDC for payments. Second, CBDC provides an alternative "safe" savings vehicle. Third, CBDC generates greater surplus in lending by reducing credit-risk information asymmetry allowing banks to offer lower loan interest rates and increase lending amounts to "good" types (and the converse to "bad" types).

We also consider the implications of an alternative "two-tier" CBDC model where non-

[2]Later, we also consider the implications of an alternative "two-tier" CBDC model where non-bank payment system providers (PSPs) can distribute CBDC.

[3]We note that the use of CBDC payments data may face political and legal hurdles in some contexts. Thus, we will also consider the implications of a CBDC where payments data cannot be used to distinguish between borrowers.

[4]Empirical evidence suggests that payment flows are informative about borrower quality (see, e.g., Mester et al., 2007; Norden and Weber, 2010; Puri et al., 2017.

bank payment system providers (PSPs) can distribute CBDC.[5] Under this model, CBDC will be used by and benefit a greater share of the population because a bank account is not required. However, the only incentive remaining for unbanked households to open a bank account, accessing CBDC through a bank over a non-bank PSP, is to establish a credit history through CBDC usage to access lower interest rate loans. Thus, less will flow from the previously unbanked into deposit accounts. Further, previously banked households may choose to save in a non-bank PSP CBDC wallet instead of a bank account. If CBDC data is shareable with the commercial bank (as may be the case under open banking), households do not have to open a bank account to build credit and access lower interest rate loans. Thus, there is a stronger incentive to open a non-bank PSP CBDC wallet, but no remaining incentive for the unbanked to open bank accounts over accessing CBDC through a non-bank PSP. Both allowing non-bank PSPs to distribute CBDC and allowing for data sharing is optimal for household welfare if the gains from greater access to CBDC and credit outweigh the contraction in lending.

We present our results both theoretically and numerically with a calibration exercise. Here, we focus on a non-interest bearing CBDC.[6] Under a set of baseline parameters calibrated to a developing economy context, we show that CBDC issuance can increase both lending and welfare. However, the lending impact can be negative if the value of CBDC as a means of payment or for credit building is small. We show that under parameters for a more developed economy context with a smaller share of the population without a bank account and/or credit history before CBDC issuance, the lending impact is more likely to be negative.

Using our baseline calibration, we also evaluate two key policy design choices for CBDC. First, we find that if the central bank chooses to disallow the use of CBDC payments data for credit building (due to privacy considerations) the positive welfare impact of CBDC falls by about a quarter. Second, we find that there is disintermediation if the central bank allows non-bank PSPs to distribute CBDC (instead of intermediation if only commercial banks distribute CBDC), but the overall welfare impact is slightly more positive from greater access to CBDC.

This paper contributes to the emerging literature on CBDCs (Infante et al., 2022; IMF, 2021; Soderberg et al., 2022; Adrian and Mancini-Griffoli, 2019; Adrian et al., 2022). Specifically, we build on the literature focused on the impact of CBDC on the banking

[5]We will explain in Section 4 that this scenario is equivalent to a "direct" (one-tier) CBDC model where the central bank directly distributes CBDC to consumers.

[6]Most countries are not considering interest-bearing CBDC.

sector (Andolfatto, 2021, Keister and Sanches, 2022, Chiu et al., 2022, Whited et al., 2022, Garratt et al., 2022, Piazzesi and Schneider, 2020, Agur et al., 2022, Chang et al., 2023). Our paper is novel in that we incorporate the implications of financial inclusion via incentivizing consumers to open bank accounts and reducing credit-risk information asymmetry in studying the potential impact of CBDC for bank lending and disintermediation.[7]

We also contribute to the literature on financial inclusion which is a crucial prerequisite to economic growth and poverty reduction in emerging market and low income economies (see Demirguc-Kunt et al., 2017). There is a strand of research that explores how CBDC can contribute to financial inclusion objectives. Auer et al. (2020) argue that while CBDCs could offer an opportunity for governments and central banks to promote universal access of financial services, they should be complemented with public policies to address other key reasons for financial exclusion. Maniff (2020) proposes several design features for a CBDC to improve financial inclusion. Wang and Hu (2022) finds that CBDCs can be useful for promoting financial inclusion only in underdeveloped economies without e-money and for strengthening financial stability in terms of curbing non-bank e-money only in developed ones.[8] Murakami et al. (2022) focuses on the monetary policy implications of CBDC providing a savings vehicle to allow unbanked households to smooth consumption.

Our paper is also related to Ahnert et al. (2022) and Brunnermeier and Payne (2022) who consider CBDC with data sharing features in different contexts. Ahnert et al. (2022) analyzes the interconnections of payments and privacy in a set up where merchants have to borrow from a bank and the bank can learn about the merchants from CBDC payment flows to extract rents. Brunnermeier and Payne (2022) studies the implications of CBDC data sharing for interoperability across digital ledger platforms.

A limitation of this paper is that we do not explicitly model issues related to weak financial literacy/capability, lack of sufficient ID, and poor internet and electricity access which should be explored further in future work. CBDC in isolation is not a panacea and may benefit from complementary public policies and private sector initiatives to address these drivers of financial exclusion.

[7] A related set of papers include an "extensive margin" decision on whether to use CBDC through a non-bank PSP in a developed country context with banks with market power (Andolfatto, 2021; Chang et al., 2023).

[8] In this paper, we do not focus on the financial stability implications of CBDC in periods of stress. In periods of financial stress, households may move funds from deposits to CBDC and it is possible that different population groups (e.g. poor vs rich, previously unbanked vs banked) may be more or less likely to do so.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 discusses results on optimal policy. Section 4 studies an alternative CBDC design where non-bank PSPs distribute CBDC. Section 5 presents results from the calibration exercise. Section 6 concludes.

# 2 Model

In this section, we present the model. In Subsection 2.1, we provide a summary. In Subsections 2.2, 2.3, and 2.4, we present the problem of the household, commercial bank, and central bank respectively.

## 2.1 Summary

There are two periods $T \in \{1, 2\}$. Households maximize utility over two periods and decide how much to save $s$ and how much to borrow $b$ for investment in a production technology. Credit markets are imperfect and there is no consumer credit (i.e. $s \geq 0$).

Households are of two types $t \in \{g, b\}$. "Good" households ($t = g$) invest successfully in a production technology with probability $p_g$ and "bad" households ($t = b$) have success with probability $p_b$, where $p_b < p_g$. Households are endowed with wealth $\omega$.

Households can open a bank account for a fixed cost $C$ which gives them access to a deposit account and CBDC wallet.

In period one ($T = 1$), households make payments for consumption, $c_1$, using CBDC with convenience value $v$ if they have a bank account or cash.[9] Households either save in a deposit account at deposit interest rate $r_d$ and liquidity risk $\ell$ (with $r_d - \ell > -d$), in a CBDC wallet at remuneration rate $r_c$, or cash at cost of storage $-d$.[10] Households also decide how much to borrow from the bank, $b$, to invest in production technology, $F(.)$, facing a loan interest rate of $r_n$ for those with no CBDC usage or $r_t$ by type $t$ for those with CBDC usage.

In period two ($T = 2$), the household is successful with probability $p_t$ and makes profits. Households consume profits and remaining savings, $c_2$, making payments using CBDC (again with convenience value $v$) if they have a bank account or cash.

[9] We assume CBDC is a valuable as a means of payment which could be derived from a range of possible features such as greater accessibility, lower cost, programmability, anonymity, network effects, or offline capabilities etc.

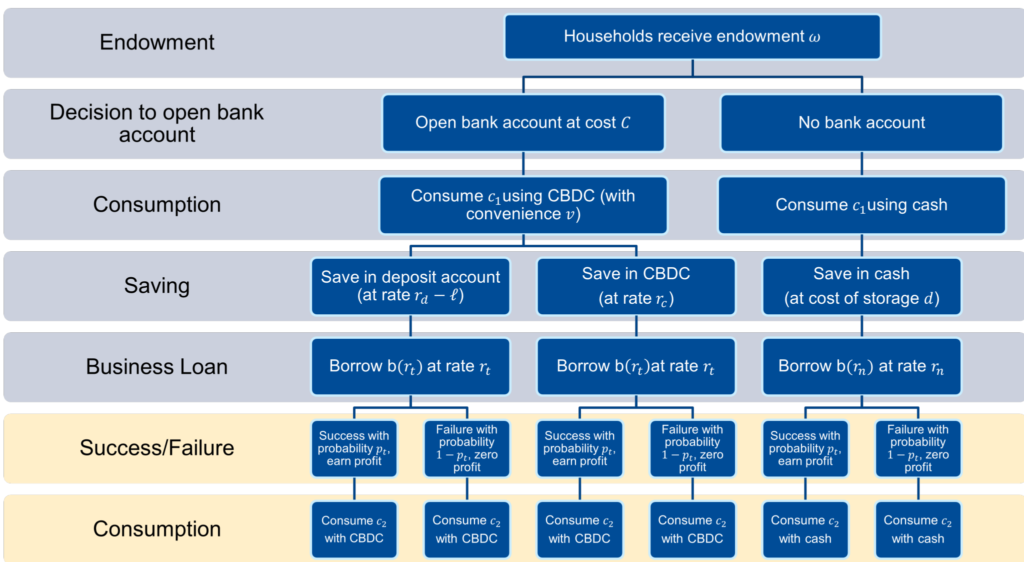
[10] Bank deposits can be thought of as saving accounts, while checking accounts for payments are abstracted from.

The commercial bank sets the deposit rate $r_d$ to attract deposits. The bank learns the household's type $t$ if the household uses CBDC for consumption. The bank sets three loan interest rates to maximize profits: (1) $r_n$ for households with no CBDC usage, (2) $r_g$ for households with CBDC usage of type $g$, and (3) $r_b$ for households with CBDC usage of type $b$. The bank is constrained by reserve requirements and is subject to free entry.

The central bank decides whether to issue CBDC and sets the CBDC remuneration rate $r_c$.

Figure 1 illustrates the model timing and sequencing.

**Figure 1:** Model Sequence




*Notes: This figure illustrates the timing and sequencing of the model presented in Section 2. For simplicity, we ignore the case where households who open bank accounts make payments in cash.*

## 2.2 Households

Households maximize expected profits over two periods. Utility function, $u(.)$, is increasing and concave, and the production technology is given by a decreasing returns to scale function, $F(.)$.

### 2.2.1 Baseline Scenario - No CBDC

We begin with a baseline scenario without CBDC. The household problem is given by:

$$U^0 = \max\{U_n, U_b\} \tag{1}$$

Households decide to open a bank account. Utility from not opening a bank account is:

$$U_n = \max_{s,b} u(c_1) + \beta(pu(c_2^s) + (1-p)u(c_2^f)) \qquad s.t. \tag{2}$$

$$c_1 = \omega - s \tag{3}$$

$$c_2^s = s(1-d) + F(b) - r_n b \tag{4}$$

$$c_2^b = s(1-d) \tag{5}$$

where $\beta$ is a time preference factor that specifies how the household trades off utility in period 1 against utility in period 2.

Utility from opening a bank account is:

$$U_b = \max_{s,b} u(c_1) + \beta(pu(c_2^s) + (1-p)u(c_2^f)) \qquad s.t. \tag{6}$$

$$c_1 = \omega - s - C \tag{7}$$

$$c_2^s = s(1 + r_d - \ell) + F(b) - r_n b \tag{8}$$

$$c_2^b = s(1 + r_d - \ell) \tag{9}$$

We show that the rich (high $\omega$) open bank accounts, while the poor (low $\omega$) do not open bank accounts. The intuition is as follows. The cost of opening a bank account $C$ is fixed, while the costs of storing cash ($-d$) and returns from the deposit interest rate ($r_d$) increase with endowment (wealth), thus the poor are unbanked.[11]

**Proposition 2.1.** *Holding all other parameters fixed, there exists a threshold $\bar{\omega}^0$ such that households open a bank account i.f.f. $\omega \geq \bar{\omega}^0$*

[11] An extensive empirical literature shows that having lower income (in addition to having a lower level of education) makes being unbanked more likely.

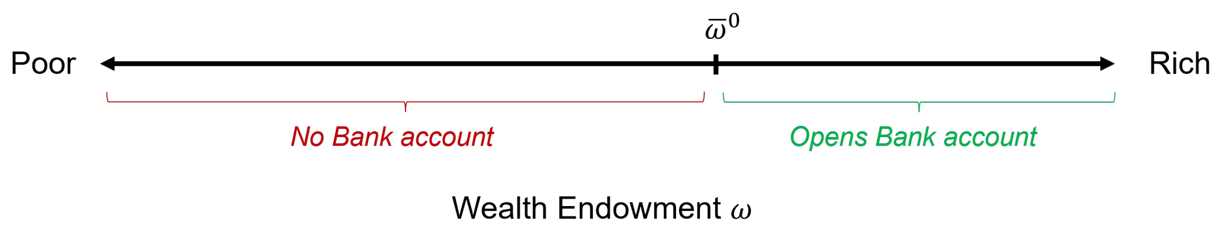



**Figure 2:** Baseline: Wealth Threshold for Bank Account

We find that poorer households open a bank account if the deposit rate $r_d$ is high, the costs of storing cash are high $d$, the liquidity risk of deposits is low $\ell$, and the cost of opening a bank account $C$ is low (Proposition A.1).

We also note that since the production technology $F(.)$ has diminishing returns to scale, lower rates imply larger loans and larger profits. This implies lower interest rates on loans increase lending and household utility (Proposition A.2).

### 2.2.2 CBDC Scenario

We introduce a CBDC and assume (for now) a two-tier model where central banks issue CBDC to commercial banks which in turn distribute them to consumers. The household problem is given by:

$$U^c = \max\{U_n, U_{b,dd}, U_{b,cd}, U_{b,dc}, U_{b,cc}\} \tag{10}$$

Households decide to open a bank account. Utility from not opening a bank account is:[12]

$$U_n = \max_{s,b} u(c_1) + \beta(pu(c_2^s) + (1-p)u(c_2^f)) \qquad s.t. \tag{11}$$

$$c_1 = \omega - s \tag{12}$$

$$c_2^s = s(1-d) + F(b) - r_n b \tag{13}$$

$$c_2^b = s(1-d) \tag{14}$$

Households decide to save in deposits or CBDC if they have a bank account. Utility from

[12]Our theoretical results would hold if we assume that households cannot borrow at all without a bank account, i.e. Equation (13) would be $c_2^s = s(1-d)$.

opening a bank account, saving in deposits, and using CBDC for payments is:

$$U_{b,dd} = \max_{s,b} u(c_1) + \beta(pu(c_2^s) + (1-p)u(c_2^f)) \qquad s.t. \tag{15}$$

$$c_1(1-v) = \omega - s - C \tag{16}$$

$$c_2^s(1-v) = s(1 + r_d - \ell) + F(b) - r_t b \tag{17}$$

$$c_2^b(1-v) = s(1 + r_d - \ell) \tag{18}$$

We note two key differences from the baseline scenario with no CBDC. First, CBDC is valuable as a means of payment with convenience value $v$, i.e. consumption is cheaper if paid for in CBDC relative to cash. This incentivizes previously unbanked households to open bank accounts to access CBDC wallets for payment. Second, households can build a credit history by using CBDC for payments. Using CBDC allows the bank to distinguish between household types, thus "good" ($g$) households face a lower interest rate $r_g$ and "bad" ($b$) households face a higher interest rate $r_b$. As a result, "good" households are incentivized to open bank accounts for CBDC payments to build credit for lower interest rates on loans.

We note that the model results are unchanged if we assume that all (rich) households with wealth $\omega$ greater than some cut-off $\Omega > \bar{\omega}^0$ already have a credit history and can access loan interest rate $r_t$. We incorporate this in the calibration exercise.

Utility from opening a bank account, saving in CBDC, and using CBDC for payments is:

$$U_{b,cd} = \max_{s,b} u(c_1) + \beta(pu(c_2^s) + (1-p)u(c_2^f)) \qquad s.t. \tag{19}$$

$$c_1(1-v) = \omega - s - C \tag{20}$$

$$c_2^s(1-v) = s(1 + r_c) + F(b) - r_t b \tag{21}$$

$$c_2^b(1-v) = s(1 + r_c) \tag{22}$$

Households can use cash for payments even if they have a bank account to avoid revealing their type $t$ to the bank. Utility from opening a bank account, saving in deposits, and using cash for payments is:

$$U_{b,dc} = \max_{s,b} u(c_1) + \beta(pu(c_2^s) + (1-p)u(c_2^f)) \qquad s.t. \tag{23}$$

$$c_1 = \omega - s - C \tag{24}$$

$$c_2^s = s(1 + r_d - \ell) + F(b) - r_n b \tag{25}$$

$$c_2^b = s(1 + r_d - \ell) \tag{26}$$

Utility from opening a bank account, saving in CBDC, and using cash for payments is:

$$U_{b,cc} = \max_{s,b} u(c_1) + \beta(pu(c_2^s) + (1-p)u(c_2^f)) \qquad s.t. \tag{27}$$

$$c_1 = \omega - s - C \tag{28}$$

$$c_2^s = s(1 + r_c) + F(b) - r_n b \tag{29}$$

$$c_2^b = s(1 + r_c) \tag{30}$$

Conditional on owning a bank account, households save in CBDC instead of deposits if the CBDC remuneration rate is greater than the deposit interest rate minus liquidity risk. This is the bank disintermediation channel where savings in deposits flow to CBDC for safety due to deposit liquidity risk.

**Proposition 2.2.** *$U_{b,dd} < U_{b,cd}$ and $U_{b,dc} < U_{b,cc}$ if and only if $r_c > r_d - \ell$*

Conditional on owning a bank account, "good" $g$-type households always make payments in CBDC instead of cash because they benefit from the value of CBDC as a means of payment and there is no risk of revealing that they are a "bad" type resulting in facing a higher loan interest rate.

**Proposition 2.3.** *$g$-type households always make payments in CBDC if $v > 0$ or $r_g < r_n$, i.e. $U_{b,dd} > U_{b,dc}$ and $U_{b,cd} > U_{b,cc}$.*

Conditional on owning a bank account, "bad" $b$-type households make payments in CBDC instead of cash if they derive sufficiently high value from CBDC as a means of payment offsetting the cost of being offered a higher loan interest rate as a result of being identified as a "bad" $b$-type household.

**Proposition 2.4.** *For b-type households, there exits a threshold $\bar{v}$ such that they make payments in CBDC, or $U_{b,dd} > U_{b,dc}$ and $U_{b,cd} > U_{b,cc}$, if and only if $v \geq \bar{v}$.*

CBDC incentivizes households who were previously unbanked to open a bank account.

This is because they want to access the value of CBDC of a means of payment $v > 0$. There is an additional incentive for "good" ($g$) households to open bank accounts as access to a CBDC wallet for payments also allows them to build a credit history and receive lower interest rate loans $r_g < r_n$. Some previously unbanked "bad" ($b$) type households may not find it worth it to open a bank account to use CBDC if the costs of revealing their type and facing a higher interest rate ($r_b > r_n$) outweighs the value of CBDC as a means of payment.

**Proposition 2.5.** *There exists thresholds $\bar{\omega}^{c,g}$ and $\bar{\omega}^{c,b}$, such that $g$-type households open bank accounts i.f.f. $\omega \geq \bar{\omega}^{c,g}$ and $b$-type households open bank accounts i.f.f. $\omega \geq \bar{\omega}^{c,b}$.*
*(1) $\bar{\omega}^{c,g} < \bar{\omega}^0$ if $v > 0$ or $r_g < r_b$.*
*(2) $\bar{\omega}^{c,b} \leq \bar{\omega}^0$*
*(3) $\bar{\omega}^{c,g} < \bar{\omega}^{c,b}$ if $r_g < r_b$.*

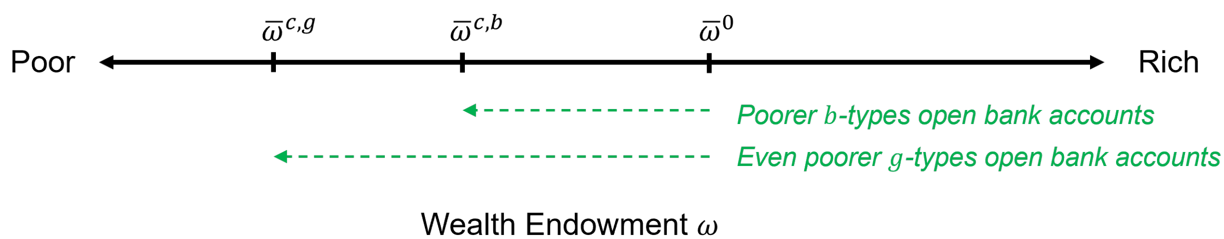



**Figure 3:** CBDC Scenario vs Baseline

We note that if banks are able to identify that a household owns a bank account and chooses not to use CBDC when making a loan, then the bank can infer that these households are "bad" types. In this case, all households will use CBDC for payment if they have a bank account as one cannot hide their type by using cash instead (Proposition A.3). "Bad" $b$-type households will have an incentive to not open a bank account to hide their type. If the gains from access to the value of CBDC as a means of payments outweighs the cost of revealing one's type, then previously unbanked households will open bank accounts. Conversely, if the cost of revealing one's type outweighs the gains from access to the value of CBDC as a means of payments, then previously banked households will close bank accounts. Note that this requires that there are enough unbanked households who are "good" types, such that pooling with the unbanked is beneficial for the "bad" types who are banked in the baseline scenario without CBDC. The impact on "bad" $b$-type households is ambiguous.

**Proposition 2.6.** *If banks are able to identify that a household owns a bank account and chooses not to use CBDC when making a loan, there exists thresholds $\bar{\omega}^{c,g}$ and $\bar{\omega}^{c,b}$, such that $g$-type*

*households open bank accounts i.f.f. $\omega \geq \bar{\omega}^{c,g}$ and b-type households open bank accounts i.f.f. $\omega \geq \bar{\omega}^{c,b}$.*
*(1) $\bar{\omega}^{c,g} < \bar{\omega}^0$ if $v > 0$ or $r_g < r_b$.*
*(2) There exists parameters such that $\bar{\omega}^{c,b} < \bar{\omega}^0$.*
*(3) There exists parameters such that $\bar{\omega}^{c,b} > \bar{\omega}^0$.*

We also consider a case where the household can choose not to allow the bank to access their CBDC data. In this case, "bad" $b$-types can always hide their type from the bank when getting a loan. Here, there is no downside from opening a bank account or using CBDC for payment. Thus, all households with a bank will use CBDC for payment (Proposition A.4). All households are incentivized to open a bank account to access the value of CBDC as a means of payment $v > 0$, and again there is an additional incentive for "good" $g$-types to open bank accounts to build credit.

**Proposition 2.7.** *If "bad" b-types can always hide their type from the bank when getting a loan (choose not to allow the bank to use their CBDC data), there exists thresholds $\bar{\omega}^{c,g}$ and $\bar{\omega}^{c,b}$, such that g-type households open bank accounts i.f.f. $\omega \geq \bar{\omega}^{c,g}$ and b-type households open bank accounts i.f.f. $\omega \geq \bar{\omega}^{c,b}$.*
*(1) $\bar{\omega}^{c,t} < \bar{\omega}^0$ if $v > 0$.*
*(2) $\bar{\omega}^{c,g} < \bar{\omega}^{c,b}$ if $r_g < r_b$.*

Poorer households open a bank account (under the CBDC scenario) if the value of CBDC as a means of payment $v$ is high, the loan interest rate for type-$t$ is low, and the CBDC remuneration rate $r_c$ is high, in addition to those described in the baseline scenario (Proposition A.5).

## 2.3 Commercial Bank

The commercial bank maximizes profits considering the utility maximization of a population of households $i$ with draws of wealth endowment $\omega(i)$, liquidity risk appetite $\ell(i)$, and type $t(i)$ from a distribution, production technology $F_i(.)$, and other fixed parameters $r_c, d, v, C$. The commercial bank is subject to a reserve requirement $m$ whereby total lending cannot exceed $m$ multiplied by total deposits.[13]

The bank sets the deposit rate $r_d$ to attract deposits. The bank learns the household's type $t$ if the household uses CBDC for consumption and sets loan interest rates, $r_n$, for those without CBDC use, $r_g$ for $g$-type households with CBDC use, and $r_b$ for $b$-type households with CBDC use.

[13]The central bank's reserve requirement on banks is exogenous. This is a simplification for tractability.

We define total deposits as

$$D(r_d) = \sum_i d_i(r_d) \tag{31}$$

where $d_i(r_d)$ is the optimal amount of savings in bank deposits at deposit interest rate $r_d$ for household $i$ with draws $\{\omega(i), \ell(i), t(i)\}$.

The commercial bank faces free-entry (competition) so profits are zero. Since the expected profit rate on a loan is $p_t r_t - r_d/m$, this implies that:[14]

$$r_n^* = \frac{r_d}{m\,\mathbb{E}[p_t|\text{no CBDC use}]} \tag{32}$$

$$r_g^* = \frac{r_d}{m p_g} \tag{33}$$

$$r_b^* = \frac{r_d}{m p_b} \tag{34}$$

Note that if higher deposit interest rates are needed to attract deposits, the commercial bank will lend less and at higher loan interest rates, i.e. deposits flowing into CBDCs (disintermediation) has an adverse affect on commercial bank lending.

We define total lending as

$$L(r_d) = \sum_i b_i(r_d) \tag{35}$$

where $b_i(r_d)$ is the optimal amount of borrowing at interest rates $r_n, r_g, r_b$ which are function of $r_d$ for household $i$ with draws $\{\omega(i), \ell(i), t(i)\}$.

Household demand for lending must not exceed total deposits multiplied by $m$.

$$L(r_d) \leq mD(r_d) \tag{36}$$

Household demand for lending is decreasing in loan interest rates $(r_n, r_g, r_b)$, while the supply of deposits is increasing in deposit interest rate $r_d$. We see that loan interest rates are tied to deposit rates by Equations 32, 33, and 34. Thus, there exists rates $\{r_d^*, r_n^*, r_g^*, r_b^*\}$ such that demand matches supply and markets clear.

[14]In practice, the central bank may choose not to allow the commercial bank to use CBDC payments data to distinguish between household types for privacy or legal reasons in some contexts. In this case, $r_n^* = r_g^* = r_b^* = \frac{r_d}{m\,\mathbb{E}[p_t]}$.

**Proposition 2.8.** *For any parameter set* $\{r_c, d, v, C\}$*, population of households* $i$ *with draws* $\{\omega(i), \ell(i), t(i)\}$ *and production technologies functions* $\{F_i\}$*, there exists prices* $\{r_d^*, r_n^*, r_g^*, r_b^*\}$ *such that all households are maximizing utility, the bank is maximizing profits, and markets clear.*

We note that the model results are identical if we incorporate wholesale funding such that the expected profit rate is $p_t r_t - \frac{(L(r_d)-W)\frac{r_d}{m}+Wr_w)}{L(r_d)}$ where $W$ is the amount of wholesale funding available at rate $r_w$. The market clearing condition in this scenario is $L(r_d) \leq mD(r_d) + W$. We incorporate this in the calibration exercise.

## 2.4 Central Bank

The central bank decides whether to issue CBDC. In the calibration exercise, we will assume that CBDC remuneration is zero, $r_c = 0$, as most countries are not considering an interest-bearing CBDC.

The central bank also sets the commercial bank reserve requirement $m$.

The central bank also decides on the “design" of the CBDC. In Section 4, we will discuss an alternative "two-tier" CBDC model where non-bank payment system providers (PSPs) can distribute CBDC.

Finally, in certain contexts, the central bank may choose not to allow the commercial bank to use CBDC payments data for credit building for privacy or legal reasons. We explore the implications of this in detail in Subsection 3.1.

# 3 Optimal Policy

We outline the conditions under which CBDC increases total lending and improves welfare.

We define the equilibrium deposit interest rate under CBDC issuance and the baseline scenario with no CBDC as $r_d^{c*}$ and $r_d^{0*}$ respectively. Note that $r_d^{c*}$ and $r_d^{0*}$ pin down the equilibrium loan interest rates $\{r_n^*, r_g^*, r_b^*\}$ by Equations 32, 33, and 34.

First, we note that issuing CBDC always increases the share of population with bank accounts (financial inclusion). This follows from Proposition 2.5.

Next, we focus on the impact of CBDC issuance on total lending. We can write the change in lending between the baseline scenario without CBDC and scenario with CBDC issuance as:

$$\Delta L = m \left[ \underbrace{\int_i s_i^c(r_d^{c*}) \mathbb{1}\{r_d^{c*} - \ell(i) \geq r_c\} \mathbb{1}\{\omega(i) \in [\bar{\omega}^{c,t(i)}, \bar{\omega}^0]\} di}_{\text{New deposits from the previously unbanked}} \right.$$

$$- \underbrace{\int_i s_i^0(r_d^{0*}) \mathbb{1}\{r_d^{c*} - \ell(i) < r_c\} \mathbb{1}\{\omega(i) \geq \bar{\omega}^0\} di}_{\text{Previously banked households saving in CBDC (Disintermediation)}}$$

$$\left. + \underbrace{\int_i (s_i^c(r_d^{c*}) - s_i^0(r_d^{0*})) \mathbb{1}\{r_d^{c*} - \ell(i) \geq r_c\} \mathbb{1}\{\omega(i) \geq \bar{\omega}^0\} di}_{\text{Change in level of savings among the previously banked}} \right] \tag{37}$$

where $s^c(r)$ is the optimal savings level at deposit interest rate $r$ under the baseline scenario with no CBDC issuance, and $s^0(r)$ is the optimal savings level at deposit interest rate $r$ under the scenario with CBDC issuance.

The impact of CBDC on total lending is the sum of: (1) the inflows of new deposits from previously unbanked households who open bank accounts in response to CBDC issuance, (2) the outflows of deposits from previously banked households who choose to save in CBDC instead of bank deposit accounts (bank disintermediation), and (3) changes in the level of savings among previously banked households, scaled by the reserve requirement. We focus on the first two drivers which are larger in magnitude.[15]

We examine the conditions under which total lending increases in response to CBDC, or $\Delta L > 0$.

First, with lower bank deposit liquidity risk, or as $\ell(i)$ for household $i$ approaches zero, less savings flow into CBDC which increases the supply of deposits for lending. This is because of lower bank disintermediation risk. The inflows of new deposits from the previously unbanked increases as more of these households who open bank accounts in response to CBDC issuance choose to save in deposit accounts instead of CBDC ($r_d^{c*} - \ell(i) \geq r_c$ for more previously unbanked households). The outflows from previously banked households saving in CBDC decreases, as more households will want to keep their savings in deposit accounts rather than move their savings to a CBDC wallet ($r_d^{c*} - \ell(i) < r_c$ for fewer previously banked households).

[15] The level of savings among previously banked households changes as the deposit interest rate moves to offset the increase or decrease in deposits from the first two drivers to clear the market.

Second, we see that the size of the deposit inflows from the previously unbanked is determined by (1) the size of the previously unbanked population, i.e. number of households with $\omega(i) \in [\bar{\omega}^{c,t}, \bar{\omega}^0]$, and (2) the level of wealth of those who open bank accounts in response to CBDC, since $s_i^c(r_d^{c*})$ is increasing in $\omega(i)$ (Lemma B.2). Thus, CBDC's impact on total lending is larger when the previously unbanked population holds more wealth (greater potential deposit flows).

Third, CBDC has a more positive impact on lending when more households are incentivized to a open bank account to access CBDC as a means of payment or to build credit. The size of the deposit inflows from the previously unbanked is larger when $\bar{\omega}^{c,t(i)}$ is lower, thus $\omega(i) \in [\bar{\omega}^{c,t(i)}, \bar{\omega}^0]$ for more households $i$. From Proposition A.5, we have that $\bar{\omega}^{c,t}$ is decreasing in $v$. Thus, when CBDC is more valuable as a means of payment, or $v$ increases, more households open bank accounts and there are greater inflows of deposits for lending. From Proposition A.5, we also have that $\bar{\omega}^{c,t}$ is increasing in $r_t$. Thus, if "good" type households are able to access lower interest rates (to earn increased profits from investment in the production technology) by building credit, or $r_n - r_g$ increases, more households open bank accounts and there are greater inflows of deposits for lending. $r_n - r_g$ is larger when the difference in probability of success between "good" and "bad" types, $p_g - p_b$, is larger which captures the extent to which CBDC allows banks to learn "more" about borrower risk profiles.

Last, we consider household welfare. Increased lending and lower interest rates imply greater aggregate levels of production and profits from Proposition A.2 and therefore greater aggregate welfare.

However, CBDC can still be optimal for household welfare when overall lending decreases. This is because although households can borrow less for investment in the production technology and earn reduced expected profits, CBDC issuance directly improves welfare through three channels.

**Proposition 3.1.** *There exists parameter set $\{r_c, d, v, C\}$, population of households $i$ with draws $\{\omega(i), \ell(i), t(i)\}$ and production technology functions $\{F_i\}$, such that $\Delta L < 0$ and aggregate welfare increases with CBDC issuance.*

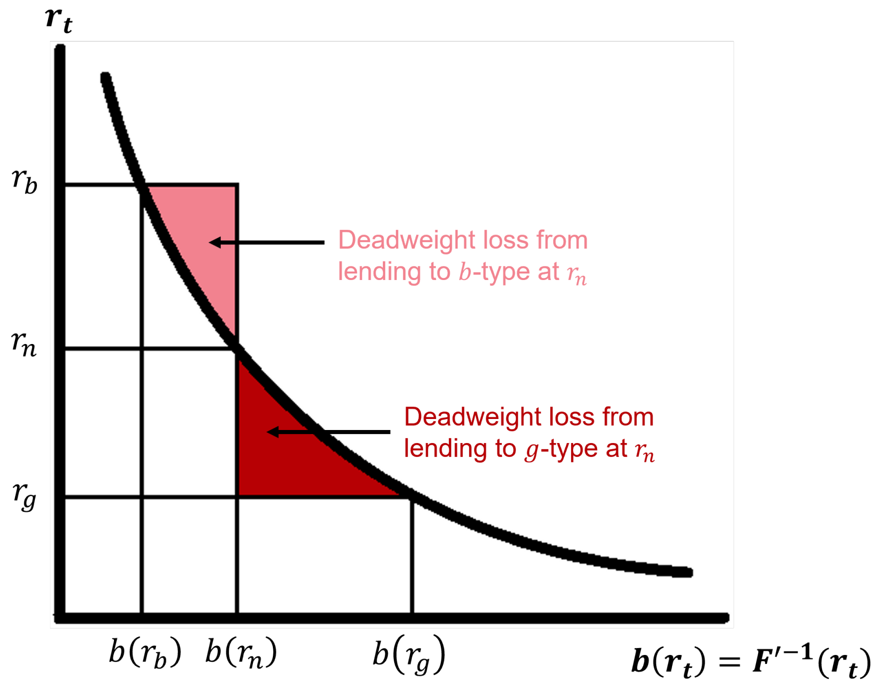
First, households gain from the value of using CBDC for payments. $v$, the value of CBDC as a means of payments, enters directly into the household utility function. Household welfare can increase even when overall lending decreases in response to CBDC issuance when $v$ is high.

Second, CBDC provides an alternative "safe" savings vehicle for those with high aversion to liquidity risk, $\ell(i)$. These households can earn $r_c$ by saving in CBDC instead of $r_d - \ell(i)$ by saving in a deposit account.

Third, CBDC reduces credit-risk information asymmetry allowing banks to offer lower loan interest rates and increase lending amounts to "good" types (and the converse to "bad" types) which generates greater aggregate household profits, boosting aggregate welfare. In the no CBDC issuance baseline scenario, all households face a pooled loan interest rate of $r_n = \frac{r_d}{m\,\mathbb{E}[p_t]}$ which is the commercial bank's expected break-even cost.[16] However, for "good" $g$-type households their true break-even cost is $r_g = \frac{r_d}{mp_g} < r_n$, thus households under-invest and miss out on potential profits, $\int_{b(r_n)}^{b(r_g)} F'(b) - r_g db$. For "bad" $b$-type households their true break-even cost is $r_b = \frac{r_d}{mp_b} > r_n$, thus households over-invest and make a loss, $\int_{b(r_b)}^{b(r_n)} r_b - F'(b)db$. Thus, by enabling banks to learn the type of households who use CBDC for payments and price based on household specific credit-risk rather than the pooled expected risk, CBDC improves social surplus. We illustrate this in Figure 4. The magnitude of this effect is larger when there is a bigger share of the population that does not have a credit history before CBDC issuance.

[16] All households also face a pooled loan interest rate of $r_n = \frac{r_d}{m\,\mathbb{E}[p_t]}$ if the commercial bank is not allowed to use CBDC payments data to distinguish between household types for credit building. Thus, there is is no welfare gains from reducing credit-risk information asymmetry.

**Figure 4:** Social Surplus and Credit-Risk Information Asymmetry




*Notes: This figure illustrates how credit-risk information asymmetry affects social surplus. The figure plots the demand curve for borrowing of a household as a function of the loan interest rate. The pink shaded area represents the deadweight loss from lending to b-type households at rate $r_n$, and the red shaded area represents the deadweight from lending to g-type households at rate $r_n$.*

## 3.1 CBDC payments data use and credit building

In certain contexts, the central bank may choose not to allow the commercial bank to use CBDC payments data for credit building. There may be significant political and legal hurdles, as idea of a public entity collecting personal data with "digital cash" could raise privacy concerns in some countries. Moreover, central banks may be subject to restrictive legislation on data sharing.

In this subsection, we discuss the implications of this for total lending and welfare.

In this case, the commercial bank is unable to use CBDC payments data to distinguish between household types. Thus, all households also face a pooled loan interest rate: $r_n^* = r_g^* = r_b^* = \frac{r_d}{m\,\mathbb{E}[p_t]}$.

Fist, disallowing the use of CBDC payments data for credit building results a more negative lending impact. From Equation 37 and as discussed above, CBDC has a more positive impact on lending when more households are incentivized to open bank accounts to access CBDC to build credit for cheaper loan interest rates. If $r_g = r_n$, there is no longer such an incentive and fewer households open bank accounts. This results in smaller inflows of deposits for lending.

Second, disallowing the use of CBDC payments data for credit building results a more negative welfare impact. Not only does lower lending and higher interest rates imply lower aggregate levels of production and profits, but the welfare gains from reducing credit-risk information asymmetry are also lost. Banks can no longer offer lower loan interest rates and increase lending amounts to "good" types (and the converse to "bad" types) which would generate greater aggregate household profits and boost aggregate welfare.

# 4 Two-tier CBDC model with non-bank PSPs

Next, we consider the implications of an alternative "two-tier" CBDC model where non-bank payment system providers (PSPs) can distribute CBDC.[17]

Let $C' < C$ be the cost of opening a CBDC wallet with a non-bank PSP. It is cheaper to open a CBDC wallet account with a non-bank PSP than a bank account. We assume for now that CBDC data cannot be shared with the commercial bank for lending.[18]

We add an additional choice of opening a PSP CBDC wallet to the CBDC scenario in Subsection 2.2.2.

The household problem is given by:

$$U^p = \max\{U_n, U_p, U_{b,dd}, U_{b,cd}, U_{b,dc}, U_{b,cc}\} \tag{38}$$

Utility from opening a CBDC wallet with a non-bank PSP, saving in CBDC, and using

[17]There may be supervisory and operational costs associated with allowing non-bank PSPs to distribute CBDC which we abstract from.

[18]We note that this scenario is equivalent to a "direct" CBDC model where the central bank also directly distributes CBDC to consumers, except that the central bank may provide access to a CBDC wallet (directly) at a cost $C'$ much closer to zero as a public good.

CBDC for payments is:

$$U_p = \max_{s,b} u(c_1) + \beta(pu(c_2^s) + (1-p)u(c_2^f)) \qquad s.t. \tag{39}$$

$$c_1(1-v) = \omega - s - C' \tag{40}$$

$$c_2^s(1-v) = s(1+r_c) + F(b) - r_n b \tag{41}$$

$$c_2^b(1-v) = s(1+r_c) \tag{42}$$

Under this design, because the fixed cost of accessing a CBDC account is cheaper through a non-bank PSP compared to a bank, more households will have access to a CBDC account and benefit from its value as a means of payment and savings vehicle.

However, fewer households will open bank accounts. Households who would have opened bank accounts to access the value of CBDC as a means of payment and savings vehicle may opt to open a non-bank PSP CBDC wallet instead. Only "good" $g$-type households will have an incentive to access CBDC through a bank over a non-bank PSP to build a credit history to gain access to lower loan interest rates.

Additionally, a non-bank PSP CBDC wallet offers an alternative savings vehicle at remuneration rate $r_c$ at a cheaper fixed cost $C' < C$, thus banked households in the baseline scenario without CBDC may opt to open a PSP CBDC wallet instead. More households may open bank accounts without CBDC than with CBDC in this scenario.

**Proposition 4.1.** *There exists thresholds $\bar{\omega}^{p1,t}$ and $\bar{\omega}^{p2,t}$ such that households of type t open a CBDC wallet with a non-bank PSP if $\bar{\omega}^{p1,t} < \omega < \bar{\omega}^{p2,t}$, open bank accounts if $\omega \geq \bar{\omega}^{p2,t}$, and do not open either if $\omega \leq \bar{\omega}^{p1,t}$.*
*(1) $\bar{\omega}^{p2,t} \geq \bar{\omega}^{c,t}$ for $t \in \{g, b\}$ (fewer households will open bank accounts).*
*(2) $\bar{\omega}^{p1,t} \leq \bar{\omega}^{c,t}$ for $t \in \{g, b\}$ (more households have access to CBDC).*
*(3) $\bar{\omega}^{p2,t} \lesseqgtr \bar{\omega}^{0}$ (Households may open non-bank PSP CBDC wallets instead).*
*(4) $\bar{\omega}^{p2,g} < \bar{\omega}^{p2,b}$ (only $g$-type HHs will have an incentive to open bank accounts to build credit).*

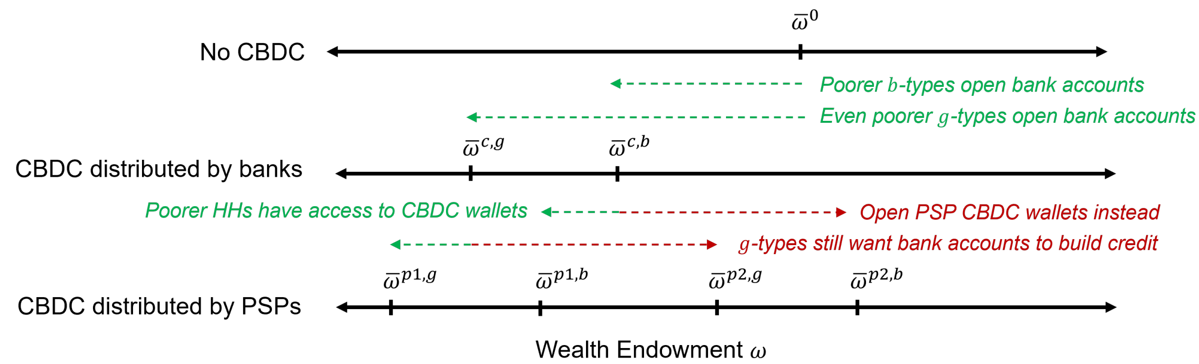



**Figure 5:** Non-bank PSP CBDC Scenario

This implies a lower level of lending compared to the traditional "two-tier" model where only commercial banks distribute CBDC (a more negative or less postive impact of CBDC on lending). However, this design could be still be optimal if the gains from greater access to the value of CBDC as a means of payments, an alternative "safe" savings vehicle and for credit building outweighs the loss in the supply of deposits and the resulting contraction in lending.

Next, we consider allowing CBDC data collected by the PSP to be shareable with the commercial bank (as may be the case under open banking). This allows households to build a credit history by using CBDC for payments through the non-bank PSP.

The household problem is given by:

$$U^s = \max\{U_n, U_p, U_{p,s}, U_{b,dd}, U_{b,cd}, U_{b,dc}, U_{b,cc}\} \tag{43}$$

We add the option to open a CBDC wallet with a non-bank PSP, save in CBDC, use CBDC for payments, and share CBDC data with the commercial bank. The utility maximization problem is:

$$U_{p,s} = \max_{s,b} u(c_1) + \beta(pu(c_2^s) + (1-p)u(c_2^f)) \qquad s.t. \tag{44}$$

$$c_1(1-v) = \omega - s - C' \tag{45}$$

$$c_2^s(1-v) = s(1+r_c) + F(b) - r_t b \tag{46}$$

$$c_2^b(1-v) = s(1+r_c) \tag{47}$$

Sharing data with the commercial bank allows the bank to identify household type $t$ and gives the household access to interest rate $r_t$. Only "good" $g$-type households will share their credit history (Proposition A.6).

If CBDC data is shareable with the commercial bank, households do not have to open a bank account to build credit or access a CBDC wallet. Fewer households may own bank accounts compared to the baseline scenario without CBDC as a non-bank PSP CBDC wallet offers an alternative savings vehicle at a cheaper fixed cost. "Good" $g$-types will have an additional incentive to open a non-bank PSP CBDC wallet to build a shareable credit history compared to the "no sharing" policy scenario.

**Proposition 4.2.** *There exists thresholds $\bar{\omega}^{s1,t}$ and $\bar{\omega}^{s2,t}$ such that households of type $t$ open a CBDC wallet with a non-bank PSP if $\bar{\omega}^{s1,t} \leq \omega < \bar{\omega}^{s2,t}$ and open bank accounts if $\omega \geq \bar{\omega}^{p2,t}$.*
*(1) $\bar{\omega}^{s1,g} < \bar{\omega}^{p1,g}$ (additional incentive for $g$-types to open PSP CBDC wallet).*
*(2) $\bar{\omega}^{s1,b} = \bar{\omega}^{p1,b}$ (no additional incentive for $b$-types).*

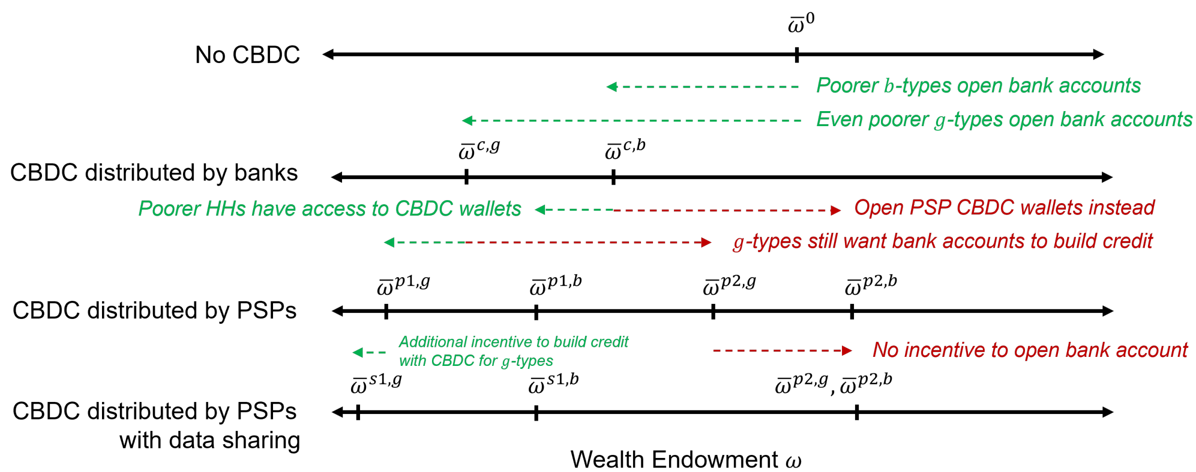



**Figure 6:** PSP with Data Sharing CBDC Scenario

Data sharing expands access to lower interest rate loans for "good" type ($g$) households who are too poor (low endowment $\omega$) to open a bank account. Allowing PSPs to distribute CBDC may be optimal for household welfare if the gains from greater access to lower interest rate loans outweigh the contraction in lending.

To summarize, issuing CBDC always increases financial inclusion with respect to payments inclusion, but financial inclusion with respect to bank account ownership depends on CBDC design. There exists a trade-off between greater payments inclusion and bank account ownership.

# 5 Calibration

In this section, we calibrate our model to that of a developing country context and estimate the impact of CBDC issuance.

We parameterize the distribution of household draws of $\{\omega(i), \ell(i), t(i)\}$ as follows. We let $\omega$ follow a Pareto distribution with shape parameter $\alpha$ and minimal and maximal values $L$ and $H$. We model the draws of $\ell$ as an exponential distribution with rate parameter $\lambda$ (where $\frac{1}{\lambda}$ is the mean). Households are type $g$ with probability $q$, and type $b$ otherwise.[19] The production technology for all households is $F_i = A_i b^{\phi}$. We let productivity be correlated with wealth: $A_i = (\omega - L)\epsilon$ where $\epsilon$ is a random uniform variable with minimal and maximal values $L'$ and $H'$.[20] We also assume that all (rich) households with $\omega$ greater than some cut off $\Omega > \bar{\omega}^0$ already have a credit history with the bank and can borrow at loan interest rate $r_t$. We calibrate the model according to parameters in Table A1, such that the share of the population with a bank account is 75%, in line with the average for developing economies according to the World Bank's Global Findex Database 2021, and the equilibrium deposit interest rate of 3%, which is typical for emerging market and low income countries. We assume CBDC remuneration is zero, $r_c = 0$, as most countries are not considering an interest-bearing CBDC.

## 5.1 Baseline Results

We present the baseline results in Table 1. Under our baseline parameters, we find that issuing CBDC increases total lending by 2.2%. This is driven by a 17 p.p. increase in the share of the population with a bank account (from 75% to 92%).[21] This offsets the bank disintermediation effect, i.e. the flow of savings from deposit accounts into CBDC wallets. 14% of bank account holders (or 13% of the overall population) choose to save in CBDC instead of in deposits. Together, the share of the population who saves in a deposit account increases 5% after CBDC issuance (from 75% to 80%) which boosts lending. 47% of the population chooses to make payments in CBDC. Aggregate profits from investing in household production technologies increases by 5%. Total household welfare (utility) increases by 0.19% from CBDC issuance.[22]

[19]For robustness, we also calibrate a version of the model where households with a larger wealth endowment are more likely to be "good" types. Here, households are type $g$ with probability $q + \delta(\omega - \frac{L+H}{2})$. We find that the results are unchanged directionally and similar in magnitude.

[20]This implies that the size of borrowing/investment is also correlated with wealth, see Proposition A.2.

[21]In line with Proposition 2.5.

[22]Our measure of welfare is the sum of household utility, where we weight all households equally.

**Table 1:** Impact of CBDC Issuance - Baseline Parameters

| | |
|---|---|
| Change in Total Lending (%) | 2.2% |
| Share of Population w/ Bank Account | 92.4% (from 75% w/o CBDC) |
| Share of Population Saving in Deposits | 79.7% |
| Share of Population Saving in a CBDC Wallet | 12.8% |
| Share of Population Making Payments with CBDC | 46.8% |
| Change in Production Profits (%) | 4.5% |
| Change in Welfare (%) | 0.19% |

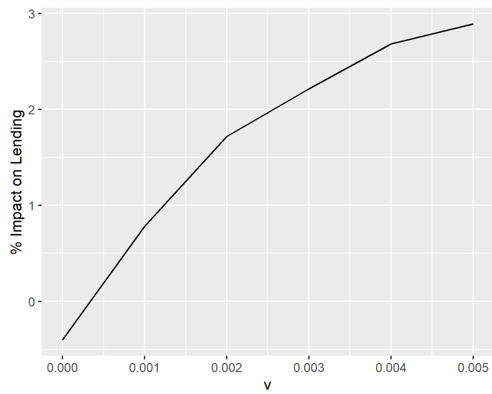
*Notes: This table presents results on the impact of CBDC issuance for a representative developing economy. The model is calibrated to the baseline parameters in Table A1.*

## 5.2 Comparative Statics

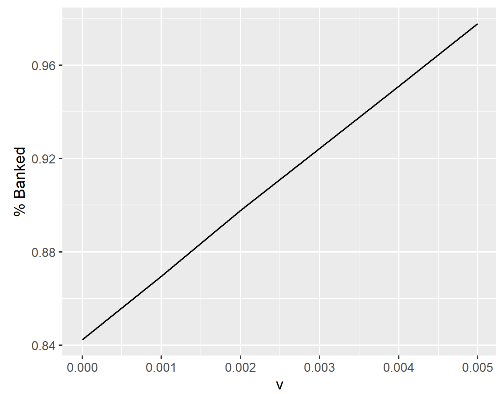
Next, we present comparative statics for key parameters in the model.

First, we consider how the impact of CBDC would vary with the value of CBDC as a means of payment, $v$. We present the results in Figure 7. The total lending impact of CBDC increases in $v$ with diminishing returns. We note that at $v = 0$ the impact on lending is negative, i.e. if there is no value of CBDC as a means of payment then CBDC issuance causes a contraction in lending. This is because fewer previously unbanked households choose to open a bank account. The share of the population who is banked and saves in deposits is increasing in $v$, as there is a stronger incentive to open a bank account. Expectedly, the share of the population using CBDC for payments increases with the value of CBDC as a means of payment ($v$). Production profits also increase in $v$ driven by the increase in investment/lending. Together, welfare is increasing in $v$. Finally, we note that although at $v = 0$ the impact of lending is negative, the welfare impact is still positive because of the value of CBDC for credit building and as an alternative savings vehicle.

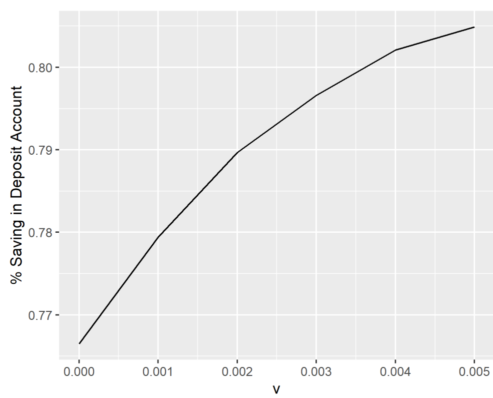
**Figure 7:** Comparative Statics: Value of CBDC as a Means of Payment ($v$)



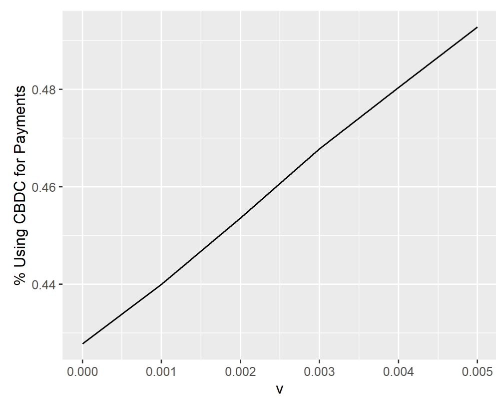

**(a)** Total Lending



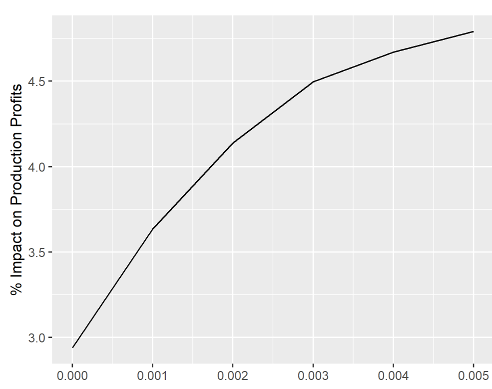

**(b)** Population with Bank Account



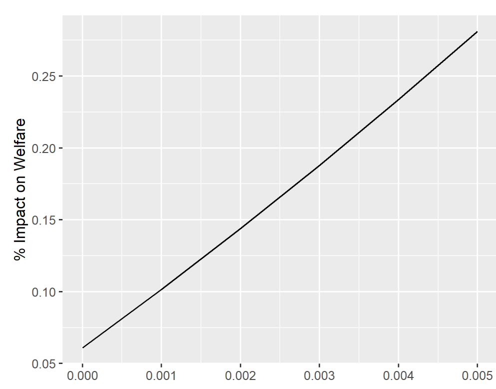

**(c)** Population Savings in Deposits




**(d)** Pop. Using CBDC for Payments




**(e)** Production Profits



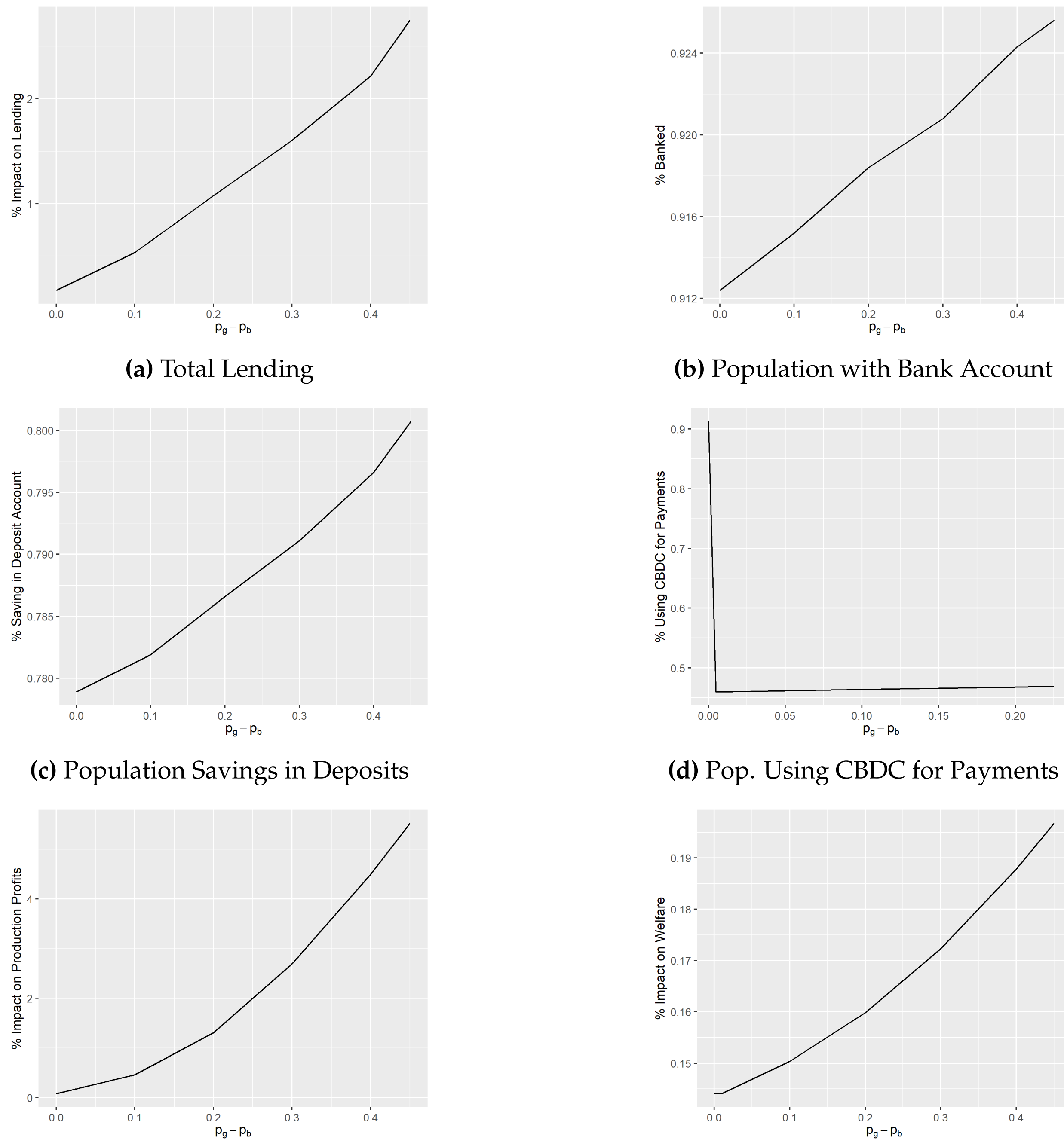

**(f)** Welfare

*Notes: These figures present the impact of CBDC issuance for different values of parameter $v$. The other parameters are fixed following Table A1.*

Next, we consider how the impact of CBDC would vary with the extent to which CBDC usage reduces credit-risk information asymmetry. We capture this in the model as the difference in probability of success between "good" and "bad" types, $p_g - p_b$. Let $p_g = p + e$ and $p_b = p - e$. We will vary $e$. If $e$ is zero, the bank learns nothing from a household's CBDC usage, if $e$ is large then there are large differences in risk profiles between households that the bank is able to identify. In Figure 8, we present the results. The total lending impact of CBDC increases in $e$. This is partially driven by the increases in the share of population with a bank account and the share saving in deposits, which are also increasing in $e$. This is because $r_n - r_g$ increases in $p_g - p_b$ so "good" type households are incentivized to open bank accounts to access lower interest rates by building credit through CBDC use. The share using CBDC for payments sharply drops after $e = 0$. This captures the fact that if $p_b \neq p_g$ then bad types have an incentive to hide their type in order to avoid higher interest rates by not using CBDC for payments. Production profits are also increasing in $e$. Greater reduction in credit-risk information asymmetry allows banks to offer lower loan interest rates and increase lending amounts to "good" types (and the converse to "bad" types) which generates greater aggregate household profits. Note that this effect is larger when there is a bigger share of the population that does not have a credit history before CBDC issuance. Together, welfare also increases in $e$.

We also discuss the case where the commercial bank is not allowed to use CBDC payments data to distinguish between household types for credit building. In this scenario, all households face a pooled loan interest rate of $r_n = \frac{r_d}{m\,\mathbb{E}[p_t]}$, as in the case where $e = 0$ above. We find that the welfare impact of CBDC issuance is 0.14%, 5 p.p. lower than the baseline.

**Figure 8:** Comparative Statics: Credit-risk Information Asymmetry ($p_g - p_b$)




**(a)** Total Lending

**(b)** Population with Bank Account

**(c)** Population Savings in Deposits

**(d)** Pop. Using CBDC for Payments

**(e)** Production Profits

**(f)** Welfare

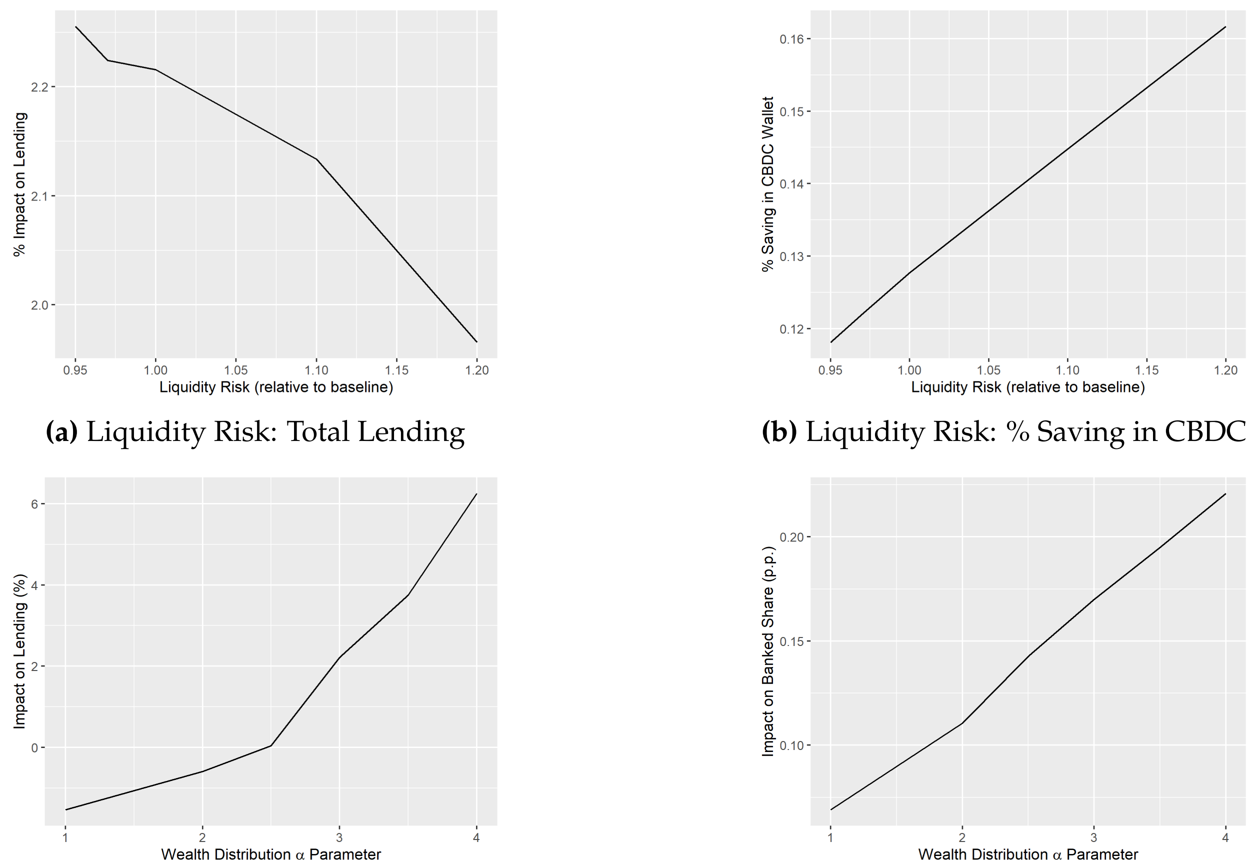
*Notes: These figures present the impact of CBDC issuance for different values of parameter e where* $p_g = p + e$ *and* $p_b = p - e$*. The other parameters are fixed following Table A1.*

Last, we consider comparative statics with respect to the population of households (instead of the features of CBDC). We present the results in Figure 9. First, we find that the impact of CBDC on total lending decreases with greater bank deposit liquidity risk, $\ell$.[23] This is because there is greater bank disintermediation, i.e. more households choose to save in CBDC. We see that the share of the population saving in CBDC increases with $\ell$. Second, we consider the household wealth distribution. Increasing the Pareto distribution parameter $\alpha$ implies a poorer population with a larger unbanked population in the baseline without CBDC.[24] As $\alpha$ increases, there are more (poor and previously unbanked) households who open bank accounts in response to CBDC. This results in the impact of CBDC on total lending to also be increasing in $\alpha$. Note that the CBDC impact on lending is negative when $\alpha$ is small. More developed and wealthier countries with smaller unbanked populations are more likely to experience a contraction in lending from CBDC issuance.

[23]We scale $\ell(i)$ upwards/downwards for all households $i$ by a constant factor.

[24]For intuition: Under the Pareto distribution, the proportion of the population whose income exceeds $x$ is $(\frac{L}{x})^{\alpha}$ which is decreasing in $\alpha$ (assuming $H = \infty$).

**Figure 9:** Comparative Statics: Liquidity Risk and Wealth Distribution




**(a)** Liquidity Risk: Total Lending

**(b)** Liquidity Risk: % Saving in CBDC

**(c)** Wealth Distribution: Total Lending

**(d)** Wealth Distribution: % Banked

*Notes: Panels (a) and (b) present the impact of CBDC issuance for liquidity risk $\ell(i) = \ell^0(i) \cdot K$ where $\ell^0(i)$ is the baseline liquidity risk and ratio K varies. Panels (c) and (d) present the impact of CBDC issuance varying the wealth Pareto distribution parameter $\alpha$. The other parameters are fixed following Table A1.*

## 5.3 Two-tier CBDC model with non-bank PSPs

Finally, we examine the trade-offs between a two-tier CBDC model where only commercial banks distribute CBDC and one where non-PSPs also distribute CBDC.

Table 2 presents the results.

We find that under a two-tier model where non-bank PSPs can distribute CBDC, the impact on total lending is negative (rather than positive if only commercial banks distribute CBDC) at -0.5% without data sharing and -1.6% with data sharing. This is because fewer households choose to open bank accounts in response to CBDC issuance. 81.2% of households have a bank account after CBDC issuance if non-bank PSPs can distribute CBDC without data sharing, compared to 92.4% if only commercial banks distribute CBDC. This is because households who would have opened bank accounts to access the value of CBDC as a means of payment and savings vehicle open a non-bank PSP CBDC wallet instead.[25] The share of households who have a bank account after CBDC issuance is even lower at 76.6% with data sharing because under that design there is no incentive to open a bank account for credit building.[26]. 17.8% of households have a non-bank PSP account without data sharing, and 22.4% do with data sharing. As a result, the share of households saving in deposits is lower at 74.4% and 73.6% without and with data sharing respectively. Lower lending implies lower production profits. Data sharing boosts production profits slightly from reducing credit risk asymmetry compared to the model without data sharing (3.5% with data-sharing versus 3.0% without).

However, under this model a greater share of households have access to CBDC for payments (61% compared to 47% if only commercial banks distribute CBDC) and as an alternative savings vehicle (22% compared to 13% if only commercial banks distribute CBDC). This offsets the welfare loss from the contraction in lending. Welfare increases by 0.20% under the a two-tier CBDC model where non-bank PSPs also can distribute CBDC without data sharing, which is 0.01 p.p. more than if only commercial banks can distribute CBDC. With data sharing, welfare increases by 0.21%, 0.01 p.p. more than without data sharing because the greater access to credit building offsets the reduced share of households who open bank accounts.

**Table 2:** Comparing CBDC Designs

| | **Only Commercial Banks** | **+ Non-bank PSPs (w/o Data Sharing)** | **+ Non-bank PSPs (w/ Data Sharing)** |
|---|---|---|---|
| Change in Total Lending (%) | 2.2% | -0.5% | -1.6% |
| % w/ Bank Account | 92.4% | 81.2% | 76.6% |
| % w/ Non-bank PSP Account | 0.0% | 17.8% | 22.4% |
| % Saving in a Bank Deposit Account | 79.7% | 74.4% | 73.6% |
| % Saving in a CBDC Wallet | 12.8% | 21.6% | 22.4% |
| % Making Payments with CBDC | 46.8% | 61.0% | 61.0% |
| Change in Production Profits (%) | 4.5% | 3.0% | 3.5% |
| Change in Welfare (%) | 0.19% | 0.20% | 0.21% |

*Notes: This table presents results on the impact of CBDC issuance for a representative developing economy for three "two-tier" CBDC designs: (1) Only commercial banks distribute CBDC, (2) Commercial banks and non-bank PSPs distribute CBDC with no data sharing, and (3) Commercial banks and non-bank PSPs distribute CBDC with data sharing. The model is calibrated to the baseline parameters in Table A1.*

[25]In line with Proposition 4.1.

[26]In line with Proposition 4.2.

# 6 Conclusion

Boosting financial inclusion is one of the main motivations for issuing retail central bank digital currencies among emerging market and low income economies.

In this paper, we develop a model incorporating the impact of financial inclusion to study the implications of introducing CBDC. Our model captures two key channels. First, CBDC issuance can increase bank deposits from the previously unbanked by incentivizing the opening of bank accounts for access to CBDC wallets (offsetting potential flows from deposits to CBDCs among those already banked). Second, CBDC usage allows for the building of credit to reduce credit-risk information asymmetry in lending.

We find that CBDC can increase overall lending if (1) bank deposit liquidity risk (disintermediation risk) is low, (2) the size and relative wealth of the previously unbanked population is large, and (3) CBDC is valuable to households as a means of payment or for credit building. CBDC can still be optimal for household welfare when overall lending decreases. This is because although households can borrow less for investment in the production technology and earn reduced expected profits, CBDC issuance directly improves welfare through three channels. First, households gain from the value of using CBDC for payments. Second, CBDC provides an alternative "safe" savings vehicle. Third, CBDC generates greater surplus in lending by reducing credit-risk information asymmetry, allowing banks to offer lower loan interest rates and increase lending amounts to "good" types (and the converse to "bad" types).

Under an alternative "two-tier" model where non-bank payment system providers (PSPs) can distribute CBDC, fewer funds will flow into deposit accounts from the unbanked because a bank account is no longer needed to access CBDC. If CBDC data is shareable with commercial banks, those without bank accounts can still build credit and access lower interest rate loans. This model is optimal for household welfare if the gains from greater access to CBDC outweigh the contraction in lending.

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# A Appendix Propositions

## A.1 Baseline Scenario – No CBDC

**Proposition A.1.** *$\bar{\omega}^0$ is increasing in C and $\ell$, and decreasing in $r_d$ and d.*

**Proposition A.2.** *Household borrowing which maximizes utility $b^*$ and household profits $F(b^*) - b^* r_n$ are decreasing in $r_n$.*

## A.2 CBDC Scenario - without non-bank PSPs

**Proposition A.3.** *If banks are able to identify that a household owns a bank account and chooses not to use CBDC when making a loan, all households always make payments in CBDC, i.e. $U_{b,dd} \geq U_{b,dc}$ and $U_{b,cd} \geq U_{b,cc}$.*

**Proposition A.4.** *If "bad" b-types can always hide their type from the bank when getting a loan (choose not to allow the bank to use their CBDC data), all households always make payments in CBDC, i.e. $U_{b,dd} \geq U_{b,dc}$ and $U_{b,cd} \geq U_{b,cc}$.*

**Proposition A.5.** *$\bar{\omega}^{c,t}$ is increasing in $r_t$, C and $\ell$, and decreasing in v, $r_c$, $r_d$ and d.*

## A.3 CBDC Scenario - with non-bank PSPs

**Proposition A.6.** *(1) $U_{p,s} > U_p$ for g-type households if $r_n > r_g$. (2) $U_{p,s} = U_p$ for b-type households.*

# B Proofs

## B.1 Proof of Proposition 2.1

**Lemma B.1.** *A solution exists that maximizes both $U_n$ and $U_b$, for $\omega > C$. The respective solutions are strictly increasing in $\omega$.*

The condition $\omega > C$ is needed as the utility maximization problem of those with a bank account is not well-defined otherwise.

The first order conditions (FOCs) of those without a bank account is given by:

$$u'(\omega - s) = (1-d)p\beta u'(s(1-d) + F(b) - r_n b) + (1-d)(1-p)\beta u'(s(1-d)) \quad (48)$$

$$F'(b) = r_n \quad (49)$$

Both the right-hand side (RHS) and left-hand side (LHS) of Equation 48 are continuous. The LHS is monotonically increasing in $s$ with $\lim_{s \to 0} u'(\omega - s) = K$ where $K$ is a finite constant and $\lim_{s \to \omega} u'(\omega - s) = \infty$. The RHS is monotonically decreasing in $s$ with $\lim_{s \to 0}(1-d)p\beta u'(s(1-d) + F(b) - r_n b) + (1-d)(1-p)\beta u'(s(1-d)) = \infty$ and $\lim_{s \to \infty}(1-d)p\beta u'(s(1-d) + F(b) - r_n b) + (1-d)(1-p)\beta u'(s(1-d)) = 0$. Continuity of both functions ensures that a solution exists and strict monotonicity ensures that it is unique. We denote the solution by $s_n(\omega)$. $s_n(\omega)$ is strictly monotonically increasing in $\omega$ because the LHS of the Equation 48 is monotonically declining in $\omega$.

The FOCs of those who open a bank account is given by:

$$u'(\omega - s - C) = (1 + r_d - \ell)p\beta u'(s(1 + r_d - \ell) + F(b) - r_n b) + (1 + r_d - \ell)(1-p)\beta u'(s(1 + r_d - \ell)) \quad (50)$$

$$F'(b) = r_n \quad (51)$$

Both the right-hand side (RHS) and left-hand side (LHS) of Equation 50 are continuous. The LHS is monotonically increasing in $s$ with $\lim_{s \to 0} u'(\omega - s - C) = K'$ where $K'$ is a finite constant and $\lim_{s \to \omega - C} u'(\omega - s - C) = \infty$. The RHS is monotonically decreasing in $s$ with $\lim_{s \to 0}(1 + r_d - \ell)p\beta u'(s(1 + r_d - \ell) + F(b) - r_n b) + (1 + r_d - \ell)(1-p)\beta u'(s(1 + r_d - \ell)) = \infty$ and $\lim_{s \to \infty}(1 + r_d - \ell)p\beta u'(s(1 + r_d - \ell) + F(b) - r_n b) + (1 + r_d - \ell)(1 - p)\beta u'(s(1 + r_d - \ell)) = 0$. Continuity of both functions ensures that a solution exists and strict monotonicity ensures that it is unique. We denote the solution by $s_b(\omega)$. $s_b(\omega)$ is strictly monotonically increasing in $\omega$ because the LHS of the Equation 48 is monotonically declining in $\omega$. □

The proof proceeds in 3 steps. First, we show that households with low $\omega$ do not open a bank account. Then, we show that a threshold $\bar{\omega}^0$ exists so that households open a bank account above that threshold. Finally, we show that if for a given $\omega_1$ households hold a bank account, they also do so for any $\omega > \omega_1$. Hence, there is a unique cutoff.

Step 1 follows immediately from $C > 0$. Poor households cannot afford to open a bank account and thus save in cash.

For step 2, consider the function $s_n(\omega)$ that solves the FOC of the unbanked. We will show that if $\omega$ is sufficiently large, households can always achieve higher utility by opening a bank account.

At the optimum, not opening a bank account achieves utility:

$$U_n = u(\omega - s_n(\omega)) + p\beta u(s_n(\omega)(1-d) + F(b^*) - b^* r_n) + (1-p)\beta u(s_n(\omega)(1-d))$$

Now, consider that the same household opens a bank account and follows savings policy $\hat{s}(\omega) = s_n(\omega) - C - \epsilon$ for $\epsilon > 0$. We will now show that it achieves higher utility than is achievable without a bank account under certain conditions. Utility for this household who opens a bank account with savings policy $\hat{s}(\omega)$ is:

$$
\begin{aligned}
U_b' &= u(\omega - C - (s_n(\omega) - C - \epsilon)) + p\beta u((s_n(\omega) - C - \epsilon)(1 + r_d - \ell) + F(b^*) - b^* r_n) + \\
&\beta(1-p)u((s_n(\omega) - C - \epsilon)(1 + r_d - \ell)) \\
&= u(\omega - s_n(\omega) + \epsilon) + p\beta u((s_n(\omega) - C - \epsilon)(1 + r_d - \ell) + F(b^*) - b^* r_n) + \\
&\beta(1-p)u((s_n(\omega) - C - \epsilon)(1 + r_d - \ell))
\end{aligned}
$$

Since $u(\omega - s_n(\omega) + \epsilon) > u(\omega - s_n(\omega))$, when $(s_n(\omega) - F - \epsilon)(1 + r_d - \ell) > s_n(\omega)(1 - d)$, the household who opens a bank account with savings policy $\hat{s}(\omega)$ achieves strictly higher utility from opening a bank account. Note that from utility maximization, we have that a household saving at $s_b(\omega)$ would achieve greater (or equal) utility than the household saving at $\hat{s}(\omega)$, i.e. $U_b \geq U_b'$.

Re-arranging implies that for any $\omega > \omega^*$ where $s_n(\omega^*) = \frac{(1+r_d-\ell)(F+\epsilon)}{r_d-\ell+d}$, opening a bank account is optimal, or $U_b > U_n$. Notice that this is only a sufficient condition. $\bar{\omega}^0 < \omega^*$ is possible.

Finally, since $s_n(\omega)$ is strictly increasing, this also implies that for any $\omega > \omega^*$, $U_b > U_n$ since $s_n(\omega) > \frac{(1+r_d-\ell)(F+\epsilon)}{r_d-\ell+d}$ for $\omega > \omega^*$. □

### B.2 Proof of Proposition A.1

$C$: For fixed $\omega$, we show that if $U_b > U_n$ for $C$, then $U_b > U_n$ if $C' < C$.

$$
\begin{aligned}
u(\omega - s_b(\omega|C') - C') + p\beta u(s_b(\omega|C')(1 + r_d - \ell) + F(b^*) - b^* r_n) + \beta(1-p)u(s_b(\omega|C')(1 + r_d - \ell)) > \\
u(\omega - s_b(\omega|C) - C') + p\beta u(s_b(\omega|C)(1 + r_d - \ell) + F(b^*) - b^* r_n) + \beta(1-p)u(s_b(\omega|C)(1 + r_d - \ell)) > \\
u(\omega - s_b(\omega|C) - C) + p\beta u(s_b(\omega|C)(1 + r_d - \ell) + F(b^*) - b^* r_n) + \beta(1-p)u(s_b(\omega|C)(1 + r_d - \ell)) > \\
u(\omega - s_n(\omega|C)) + p\beta u(s_n(\omega|C)(1-d) + F(b^*) - b^* r_n) + (1-p)\beta u(s_n(\omega|C)(1-d)) = \\
u(\omega - s_n(\omega|C')) + p\beta u(s_n(\omega|C')(1-d) + F(b^*) - b^* r_n) + (1-p)\beta u(s_n(\omega|C')(1-d))
\end{aligned}
$$

The first inequality follows from utility maximization given $C'$, the second inequality follows from $U' > 0$, the third inequality follows from $U_b > U_n$ for $C$, and the fourth equality follows from the fact that $C$ does not enter the utility function for the unbanked.

$\ell$: For fixed $\omega$, we show that if $U_b > U_n$ for $\ell$, then $U_b > U_n$ if $\ell' < \ell$.

$$\begin{aligned}
u(\omega - s_b(\omega|\ell') - C) + p\beta u(s_b(\omega|\ell')(1 + r_d - \ell') + F(b^*) - b^* r_n) + \beta(1-p)u(s_b(\omega|\ell')(1 + r_d - \ell')) > \\
u(\omega - s_b(\omega|\ell) - C) + p\beta u(s_b(\omega|\ell)(1 + r_d - \ell') + F(b^*) - b^* r_n) + \beta(1-p)u(s_b(\omega|\ell')(1 + r_d - \ell)) > \\
u(\omega - s_b(\omega|\ell) - C) + p\beta u(s_b(\omega|\ell)(1 + r_d - \ell) + F(b^*) - b^* r_n) + \beta(1-p)u(s_b(\omega|\ell)(1 + r_d - \ell)) > \\
u(\omega - s_n(\omega|\ell)) + p\beta u(s_n(\omega|\ell)(1-d) + F(b^*) - b^* r_n) + (1-p)\beta u(s_n(\omega|\ell)(1-d)) = \\
u(\omega - s_n(\omega|\ell')) + p\beta u(s_n(\omega|\ell')(1-d) + F(b^*) - b^* r_n) + (1-p)\beta u(s_n(\omega|\ell')(1-d))
\end{aligned}$$

The first inequality follows from utility maximization given $\ell'$, the second inequality follows from $U' > 0$, the third inequality follows from $U_b > U_n$ for $\ell$, and the fourth equality follows from the fact that $\ell$ does not enter the utility function for the unbanked.

$r_d$: For fixed $\omega$, we show that if $U_b > U_n$ for $r_d$, then $U_b > U_n$ if $r'_d > r_d$.

$$\begin{aligned}
u(\omega - s_b(\omega|r'_d) - C) + p\beta u(s_b(\omega|r'_d)(1 + r'_d - \ell) + F(b^*) - b^* r_n) + \beta(1-p)u(s_b(\omega|r'_d)(1 + r'_d - \ell)) > \\
u(\omega - s_b(\omega|r_d) - C) + p\beta u(s_b(\omega|r_d)(1 + r'_d - \ell) + F(b^*) - b^* r_n) + \beta(1-p)u(s_b(\omega|r_d)(1 + r'_d - \ell)) > \\
u(\omega - s_b(\omega|r_d) - C) + p\beta u(s_b(\omega|r_d)(1 + r_d - \ell) + F(b^*) - b^* r_n) + \beta(1-p)u(s_b(\omega|r_d)(1 + r_d - \ell)) > \\
u(\omega - s_n(\omega|r_d)) + p\beta u(s_n(\omega|r_d)(1-d) + F(b^*) - b^* r_n) + (1-p)\beta u(s_n(\omega|r_d)(1-d)) = \\
u(\omega - s_n(\omega|r'_d)) + p\beta u(s_n(\omega|r'_d)(1-d) + F(b^*) - b^* r_n) + (1-p)\beta u(s_n(\omega|r'_d)(1-d))
\end{aligned}$$

The first inequality follows from utility maximization given $r'_d$, the second inequality follows from $U' > 0$, the third inequality follows from $U_b > U_n$ for $r_d$, and the fourth equality follows from the fact that $r_d$ does not enter the utility function for the unbanked.

$d$: For fixed $\omega$, we show that if $U_b < U_n$ for $d$, then $U_b < U_n$ if $d' < d$.

$$\begin{aligned}
u(\omega - s_b(\omega|d') - C) + p\beta u(s_b(\omega|d')(1 + r_d - \ell) + F(b^*) - b^* r_n) + \beta(1-p)u(s_b(\omega|d')(1 + r_d - \ell)) = \\
u(\omega - s_b(\omega|d) - C) + p\beta u(s_b(\omega|d)(1 + r_d - \ell) + F(b^*) - b^* r_n) + \beta(1-p)u(s_b(\omega|d)(1 + r_d - \ell)) < \\
u(\omega - s_n(\omega|d)) + p\beta u(s_n(\omega|d)(1-d) + F(b^*) - b^* r_n) + (1-p)\beta u(s_n(\omega|d)(1-d)) < \\
u(\omega - s_n(\omega|d)) + p\beta u(s_n(\omega|d)(1-d') + F(b^*) - b^* r_n) + (1-p)\beta u(s_n(\omega|d)(1-d')) < \\
u(\omega - s_n(\omega|d')) + p\beta u(s_n(\omega|d')(1-d') + F(b^*) - b^* r_n) + (1-p)\beta u(s_n(\omega|d')(1-d'))
\end{aligned}$$

The first equality follows from the fact that $d$ does not enter the utility function for the banked, the second inequality follows from $U_b < U_n$ for $d$, the third inequality follows from $U' > 0$, the last inequality follows from utility maximization given $d'$. $\square$

### B.3 Proof of Proposition A.2

Rearranging Equation 51 gives us $b^* = F'^{-1}(r_n)$. Since $F$ is a diminishing returns function, $b^*$ is decreasing in $r_n$.

Next, we prove that $F(b^*) - b^* r_n$ is decreasing in $r_n$. Let $r_n' < r_n$ and $b'^* = F'^{-1}(r_n')$, we show that $F(b^*) - b^* r_n < F(b'^*) - b'^* r_n'$.

$$F(b^*) - b^* r_n < F(b^*) - b^* r_n' < F(b'^*) - b'^* r_n'$$

The first inequality follows from $r_n' < r_n$, and the second inequality follows from profit maximization given $r_n'$. □

### B.4 Proof of Proposition 2.2

Let $r_c > r_d - \ell$. Define $s^*$ as the optimal saving for saving in deposits and $s^{**}$ as the optimal saving for saving in cash (conditional on opening bank account and using cash for payments).

$$
\begin{aligned}
&U_{b,cc} = u(\omega - s^{**} - C) + \beta(pu(s^{**}(1+r_c) + F(b^*) - r_n b^*) + (1-p)u(s^{**}(1+r_c))) > \\
&u(\omega - s^* - C) + \beta(pu(s^*(1+r_c) + F(b^*) - r_n b^*) + (1-p)u(s^*(1+r_c))) > \\
&u(\omega - s^* - C) + \beta(pu(s^*(1+r_d-\ell) + F(b^*) - r_n b^*) + (1-p)u(s^*(1+r_d-\ell))) = U_{b,dc}
\end{aligned}
$$

The first inequality comes from utility maximization, and the second inequality comes from $r_c > r_d - \ell$.

Now let $r_c < r_d - \ell$.

$$
\begin{aligned}
&U_{b,dc} = u(\omega - s^* - C) + \beta(pu(s^*(1+r_d-\ell) + F(b^*) - r_n b^*) + (1-p)u(s^*(1+r_d-\ell))) > \\
&u(\omega - s^{**} - C) + \beta(pu(s^{**}(1+r_d-\ell) + F(b^*) - r_n b^*) + (1-p)u(s^{**}(1+r_d-\ell))) > \\
&u(\omega - s^{**} - C) + \beta(pu(s^{**}(1+r_c) + F(b^*) - r_n b^*) + (1-p)u(s^{**}(1+r_c))) = U_{b,cc}
\end{aligned}
$$

The first inequality comes from utility maximization, and the second inequality comes from $r_c < r_d - \ell$.

The same proof follows for $U_{b,dd}$ and $U_{b,cd}$ (the using CBDC for payments case). □

## B.5 Proof of Proposition 2.3

Define $s^*$ as the optimal saving for paying in CBDC and $s^{**}$ as the optimal saving for paying in cash (conditional on opening bank account and saving in CBDC). Let $\pi(r) = \max_b F(b) - br$, note that $\pi(r_n) < \pi(r_g)$ i.f.f. $r_n > r_g$ from Proposition A.2.

$$\begin{aligned} U_{b,cd} = u(\frac{\omega - s^* - C}{1-v}) + \beta(pu(\frac{s^*(1+r_c) + \pi(r_g)}{1-v}) + (1-p)u(\frac{s^*(1+r_c)}{1-v})) > \\ u(\frac{\omega - s^{**} - C}{1-v}) + \beta(pu(\frac{s^{**}(1+r_c) + \pi(r_g)}{1-v}) + (1-p)u(\frac{s^{**}(1+r_c)}{1-v})) > \\ u(\omega - s^{**} - C) + \beta(pu(s^{**}(1+r_c) + \pi(r_n)) + (1-p)u(s^{**}(1+r_c))) = U_{b,cc} \end{aligned}$$

The first inequality follows from utility maximization, and the second inequality follows from $\pi(r_g) > \pi(r_n)$ if $r_g < r_n$ or $v > 0$.

The same proof follows for $U_{b,dc}$ and $U_{b,dd}$ (the saving in deposits case). □

## B.6 Proof of Proposition 2.4

Let $\pi(r) = \max_b F(b) - br$, note that $\pi(r_n) < \pi(r_g)$ i.f.f. $r_n > r_g$ from Proposition A.2.

Define $\bar{v}$ such that

$$\begin{aligned} U_{b,cd} = u(\frac{\omega - s - C}{1-\bar{v}}) + \beta(pu(\frac{s(1+r_c) + \pi(r_b)}{1-\bar{v}}) + (1-p)u(\frac{s(1+r_c)}{1-\bar{v}})) = \\ u(\omega - s - C) + \beta(pu(s(1+r_c) + \pi(r_n)) + (1-p)u(s(1+r_c))) = U_{b,cc} \end{aligned}$$

$\bar{v}$ exists since $U_{b,cd}$ is continuous and monotonically increasing in $v$ and $U_{b,cd}$ is constant in $v$. Then if $v > \bar{v}$

$$\begin{aligned} U_{b,cd} = u(\frac{\omega - s - C}{1-v}) + \beta(pu(\frac{s(1+r_c) + \pi(r_b)}{1-v}) + (1-p)u(\frac{s(1+r_c)}{1-v})) > \\ u(\frac{\omega - s - C}{1-\bar{v}}) + \beta(pu(\frac{s(1+r_c) + \pi(r_b)}{1-\bar{v}}) + (1-p)u(\frac{s(1+r_c)}{1-\bar{v}})) = \\ u(\omega - s - C) + \beta(pu(s(1+r_c) + \pi(r_n)) + (1-p)u(s(1+r_c))) = U_{b,cc} \end{aligned}$$

since $u(.)$ is increasing.

Also, if $v < \bar{v}$

$$\begin{aligned} U_{b,cd} = u(\frac{\omega - s - C}{1-v}) + \beta(pu(\frac{s(1+r_c)+\pi(r_b)}{1-v}) + (1-p)u(\frac{s(1+r_c)}{1-v})) < \\ u(\frac{\omega - s - C}{1-\bar{v}}) + \beta(pu(\frac{s(1+r_c)+\pi(r_b)}{1-\bar{v}}) + (1-p)u(\frac{s(1+r_c)}{1-\bar{v}})) = \\ u(\omega - s - C) + \beta(pu(s(1+r_c)+\pi(r_n)) + (1-p)u(s(1+r_c))) = U_{b,cc} \end{aligned}$$

The same proof follows for $U_{b,dc}$ and $U_{b,dd}$ (the saving in deposits case). □

## B.7 Proof of Proposition 2.5

There exists thresholds $\bar{\omega}^{c,g}$ and $\bar{\omega}^{c,b}$ following the same proof concept as in Proposition 2.1. It remains to show that:

(1) $\bar{\omega}^{c,g} < \bar{\omega}^0$ if $v > 0$ or $r_g < r_b$.

(2) $\bar{\omega}^{c,b} \leq \bar{\omega}^0$

(3) $\bar{\omega}^{c,g} < \bar{\omega}^{c,b}$ if $r_g < r_b$.

Let $s^{b,dd}$ be the optimal saving for households with a bank account and save in deposits in the CBDC scenario, and $s^b$ be the optimal saving for households with a bank account in the baseline scenario without CBDC, and $s^n$ be the optimal saving for the unbanked. Let $\pi(r) = \max_b F(b) - br$, note that $\pi(r_n) < \pi(r_g)$ i.f.f. $r_n > r_g$ from Proposition A.2.

Let $\omega = \bar{\omega}^0$. If the household is type $g$ and saves in deposits:

$$\begin{aligned} U_{b,dd} = u(\frac{\bar{\omega}^0 - s^{b,dd} - C}{1-v}) + \beta(pu(\frac{s^{b,dd}(1+r_d-\ell)+\pi(r_g)}{1-v}) + (1-p)u(\frac{s^{b,dd}(1+r_d-\ell)}{1-v})) > \\ u(\frac{\bar{\omega}^0 - s^b - C}{1-v}) + \beta(pu(\frac{s^b(1+r_d-\ell)+\pi(r_g)}{1-v}) + (1-p)u(\frac{s^b(1+r_d-\ell)}{1-v})) > \\ u(\bar{\omega}^0 - s^b - C) + \beta(pu(s^b(1+r_d-\ell)+\pi(r_n)) + (1-p)u(s^b(1+r_d-\ell))) = \\ u(\bar{\omega}^0 - s^n) + \beta(pu(s^n(1-d)+\pi(r_n)) + (1-p)u(s^n(1-d))) = U_n \end{aligned}$$

The first inequality holds from utility maximization, the second inequality holds if $v > 0$ or $r_g < r_b$, and the last equality holds since $U_b = U_n$ at $\bar{\omega}^0$.

If the household is type $g$ and saves in CBDC, then $U_{b,cd} > U_{b,dd}$ so $U_{b,cd} > U_{b,n}$.

Thus, $U_{b,cd}, U_{b,dd} > U_n$ at $\omega = \bar{\omega}^0$. This implies $\bar{\omega}^0 > \bar{\omega}^{c,g}$, proving (1).

If the household is type $b$ and chooses to use cash for payments:

$$U_{b,dc} = u(\bar{\omega}^0 - s^* - C) + \beta(pu(s^*(1+r_d-\ell)+\pi(r_n)) + (1-p)u(s^*(1+r_d-\ell))) =$$
$$u(\bar{\omega}^0 - s^{**} - C) + \beta(pu(s^{**}(1+r_d-\ell)+\pi(r_n)) + (1-p)u(s^{**}(1+r_d-\ell))) =$$
$$u(\bar{\omega}^0 - s^{**}) + \beta(pu(s^{**}(1-d)+\pi(r_n)) + (1-p)u(s^{**}(1-d))) = U_n$$

The first inequality holds since the utility maximization problem, the second equality holds since $U_b = U_n$ at $\bar{\omega}^0$.

Thus, $\min\{U_{b,dd}, U_{b,cd}, U_{b,dc}, U_{b,cc}\} \geq U_n$, so $\bar{\omega}^0 \geq \bar{\omega}^{c,b}$, proving (2).

Now, we prove (3) and let $\omega = \bar{\omega}^{c,b}$.

Let utility for type $t$ household be denoted by $U_{(.)}(t)$ and savings policy for type $t$ household be denoted by $s^{(.)}(t)$.

$$U_{b,dd}(g) = u(\frac{\bar{\omega}^{c,b} - s^{b,dd}(g) - C}{1-v}) + \beta(pu(\frac{s^{b,dd}(g)(1+r_d-\ell)+\pi(r_g)}{1-v}) + (1-p)u(\frac{s^{b,dd}(g)(1+r_d-\ell)}{1-v})) >$$
$$u(\frac{\bar{\omega}^{c,b} - s^{b,dd}(b) - C}{1-v}) + \beta(pu(\frac{s^{b,dd}(b)(1+r_d-\ell)+\pi(r_g)}{1-v}) + (1-p)u(\frac{s^{b,dd}(b)(1+r_d-\ell)}{1-v})) >$$
$$u(\frac{\bar{\omega}^{c,b} - s^{b,dd}(b) - C}{1-v}) + \beta(pu(\frac{s^{b,dd}(b)(1+r_d-\ell)+\pi(r_b)}{1-v}) + (1-p)u(\frac{s^{b,dd}(b)(1+r_d-\ell)}{1-v})) = U_{b,dd}(b)$$

The first inequality holds from utility maximization and the second inequality holds from $\pi(r_b) < \pi(r_g)$ since $r_b > r_g$.

Similarly, we can show $U_{b,cd}(g) > U_{b,cd}(b)$.

$U_{b,cc}(b) = U_{b,cc}(g)$ and $U_{b,dc}(b) = U_{b,dc}(g)$ since $r_t$ does not enter into the utility maximization problem. Also, from Proposition 2.3 we have $U_{b,dd} > U_{b,dc}$ and $U_{b,cd} > U_{b,cc}$.

Together we have

$$\max\{U_{b,dd}(g), U_{b,cd}(g), U_{b,cc}(g), U_{b,dc}(g)\} > \max\{U_{b,dd}(b), U_{b,cd}(b), U_{b,cc}(b), U_{b,dc}(b)\} = U_n$$

where the last equality holds because $\omega = \bar{\omega}^{c,b}$.

Thus, $U_{b,cd}, U_{b,dd} > U_n$ at $\omega = \bar{\omega}^{c,b}$. This implies $\bar{\omega}^{c,b} > \bar{\omega}^{c,g}$, proving (3).

## B.8 Proof of Proposition A.3

Define $s^*$ as the optimal saving for paying in CBDC and $s^{**}$ as the optimal saving for paying in cash (conditional on opening bank account and saving in CBDC). Let $\pi(r) = \max_b F(b) - br$, note that $\pi(r_n) < \pi(r_g)$ i.f.f. $r_n > r_g$ from Proposition A.2.

$$\begin{aligned} U_{b,cd} = u(\frac{\omega - s^* - C}{1-v}) + \beta(pu(\frac{s^*(1+r_c)+\pi(r_t)}{1-v}) + (1-p)u(\frac{s^*(1+r_c)}{1-v})) \geq \\ u(\frac{\omega - s^{**} - C}{1-v}) + \beta(pu(\frac{s^{**}(1+r_c)+\pi(r_t)}{1-v}) + (1-p)u(\frac{s^{**}(1+r_c)}{1-v})) \geq \\ u(\omega - s^{**} - C) + \beta(pu(s^{**}(1+r_c)+\pi(r_t)) + (1-p)u(s^{**}(1+r_c))) = U_{b,cc} \end{aligned}$$

The first inequality follows from utility maximization, and the second inequality follows from $v \geq 0$.

The same proof follows for $U_{b,dc}$ and $U_{b,dd}$ (the saving in deposits case). □

## B.9 Proof of Proposition 2.6

There exists thresholds $\bar{\omega}^{c,g}$ and $\bar{\omega}^{c,b}$ following the same proof concept as in Proposition 2.1.

(1) follows from same proof concept of (1) in Proposition 2.5.

To show (2), consider $v > 0$ and $r_g = r_b = r_n$.

Let $s^{b,dd}$ be the optimal saving for households with a bank account and save in deposits in the CBDC scenario, and $s^b$ be the optimal saving for households with a bank account in the baseline scenario without CBDC, and $s^n$ be the optimal saving for the unbanked. Let $\pi(r) = \max_b F(b) - br$, note that $\pi(r_n) < \pi(r_g)$ i.f.f. $r_n > r_g$ from Proposition A.2.

Let $\omega = \bar{\omega}^0$. If the household is type $t$ and saves in deposits:

$$\begin{aligned} U_{b,dd} = u(\frac{\bar{\omega}^0 - s^{b,dd} - C}{1-v}) + \beta(pu(\frac{s^{b,dd}(1+r_d-\ell)+\pi(r_t)}{1-v}) + (1-p)u(\frac{s^{b,dd}(1+r_d-\ell)}{1-v})) > \\ u(\frac{\bar{\omega}^0 - s^b - C}{1-v}) + \beta(pu(\frac{s^b(1+r_d-\ell)+\pi(r_t)}{1-v}) + (1-p)u(\frac{s^b(1+r_d-\ell)}{1-v})) > \\ u(\bar{\omega}^0 - s^b - C) + \beta(pu(s^b(1+r_d-\ell)+\pi(r_n)) + (1-p)u(s^b(1+r_d-\ell))) = \\ u(\bar{\omega}^0 - s^n) + \beta(pu(s^n(1-d)+\pi(r_n)) + (1-p)u(s^n(1-d))) = U_n \end{aligned}$$

The first inequality holds from utility maximization, the second inequality holds if $v > 0$, and the last equality holds since $U_b = U_n$ at $\bar{\omega}^0$.

If the household saves in CBDC, then $U_{b,cd} > U_{b,dd}$ so $U_{b,cd} > U_{b,n}$.

Thus, $U_{b,cd}, U_{b,dd} > U_n$ at $\omega = \bar{\omega}^0$. This implies $\bar{\omega}^0 > \bar{\omega}^{c,b}$, proving (2).

To show (3), consider $v = 0$, and $r_g < r_n < r_b$. Also, let $r_d - \ell > r_c$.

Let $s^{b,dd}$ be the optimal saving for households with a bank account and save in deposits in the CBDC scenario, and $s^b$ be the optimal saving for households with a bank account in the baseline scenario without CBDC, and $s^n$ be the optimal saving for the unbanked. Let $\pi(r) = \max_b F(b) - br$, note that $\pi(r_n) < \pi(r_g)$ i.f.f. $r_n > r_g$ from Proposition A.2.

Let $\omega = \bar{\omega}^0$. If the household is type $t$ and saves in deposits:

$$\begin{aligned}
U_{b,dd} = u(\frac{\bar{\omega}^0 - s^{b,dd} - C}{1-v}) + \beta(pu(\frac{s^{b,dd}(1+r_d-\ell)+\pi(r_b)}{1-v}) + (1-p)u(\frac{s^{b,dd}(1+r_d-\ell)}{1-v})) < \\
u(\frac{\bar{\omega}^0 - s^{b,dd} - C}{1-v}) + \beta(pu(\frac{s^{b,dd}(1+r_d-\ell)+\pi(r_n)}{1-v}) + (1-p)u(\frac{s^{b,dd}(1+r_d-\ell)}{1-v})) = \\
u(\bar{\omega}^0 - s^{b,dd} - C) + \beta(pu(s^{b,dd}(1+r_d-\ell)+\pi(r_n)) + (1-p)u(s^{b,dd}(1+r_d-\ell))) < \\
u(\bar{\omega}^0 - s^b - C) + \beta(pu(s^b(1+r_d-\ell)+\pi(r_n)) + (1-p)u(s^b(1+r_d-\ell))) = \\
u(\bar{\omega}^0 - s^n) + \beta(pu(s^n(1-d)+\pi(r_n)) + (1-p)u(s^n(1-d))) = U_n
\end{aligned}$$

The first inequality holds since $r_b > r_n$, the second equality holds since $v = 0$, the third inequality holds because of utility maximization, and the last equality holds because $U_b = U_n$ at $\bar{\omega}^0$

Thus, $U_{b,dd} < U_n$ at $\omega = \bar{\omega}^0$. This implies $\bar{\omega}^0 < \bar{\omega}^{c,b}$, proving (3). □

## B.10 Proof of Proposition A.4

Define $s^*$ as the optimal saving for paying in CBDC and $s^{**}$ as the optimal saving for paying in cash (conditional on opening bank account and saving in CBDC). Let $\pi(r) = \max_b F(b) - br$, note that $\pi(r_n) < \pi(r_g)$ i.f.f. $r_n > r_g$ from Proposition A.2.

For type $g$:

$$U_{b,cd} = u(\frac{\omega - s^* - C}{1-v}) + \beta(pu(\frac{s^*(1+r_c) + \pi(r_g)}{1-v}) + (1-p)u(\frac{s^*(1+r_c)}{1-v})) \geq$$
$$u(\frac{\omega - s^{**} - C}{1-v}) + \beta(pu(\frac{s^{**}(1+r_c) + \pi(r_g)}{1-v}) + (1-p)u(\frac{s^{**}(1+r_c)}{1-v})) \geq$$
$$u(\omega - s^{**} - C) + \beta(pu(s^{**}(1+r_c) + \pi(r_n)) + (1-p)u(s^{**}(1+r_c))) = U_{b,cc}$$

The first inequality follows from utility maximization, and the second inequality follows from $v > 0$ and $r_g \leq r_n$ (which implies $\pi(r_n) \leq \pi(r_g)$).

For type $b$:

$$U_{b,cd} = u(\frac{\omega - s^* - C}{1-v}) + \beta(pu(\frac{s^*(1+r_c) + \pi(r_n)}{1-v}) + (1-p)u(\frac{s^*(1+r_c)}{1-v})) \geq$$
$$u(\frac{\omega - s^{**} - C}{1-v}) + \beta(pu(\frac{s^{**}(1+r_c) + \pi(r_n)}{1-v}) + (1-p)u(\frac{s^{**}(1+r_c)}{1-v})) \geq$$
$$u(\omega - s^{**} - C) + \beta(pu(s^{**}(1+r_c) + \pi(r_n)) + (1-p)u(s^{**}(1+r_c))) = U_{b,cc}$$

The first inequality follows from utility maximization, and the second inequality follows from $v > 0$.

The same proof follows for $U_{b,dc}$ and $U_{b,dd}$ (the saving in deposits case). □

## B.11 Proof of Proposition 2.7

This proof follows the same proof concept of Proposition 2.5.

## B.12 Proof of Proposition A.5

$v$: For fixed $\omega$, we show that if $U_{b,dd} > U_n$ for $v$, then $U_{b,dd} > U_n$ if $v' > v$.

$$u(\frac{\omega - s_{b,dd}(\omega|v') - C}{1-v'}) + p\beta u(\frac{s_{b,dd}(\omega|v')(1+r_d-\ell) + F(b^*) - b^* r_t}{1-v'}) + \beta(1-p)u(\frac{s_{b,dd}(\omega|v')(1+r_d-\ell)}{1-v'}) >$$
$$u(\frac{\omega - s_{b,dd}(\omega|v) - C}{1-v'}) + p\beta u(\frac{s_{b,dd}(\omega|v)(1+r_d-\ell) + F(b^*) - b^* r_t}{1-v'}) + \beta(1-p)u(\frac{s_{b,dd}(\omega|v)(1+r_d-\ell)}{1-v'}) >$$
$$u(\frac{\omega - s_{b,dd}(\omega|v) - C}{1-v}) + p\beta u(\frac{s_{b,dd}(\omega|v)(1+r_d-\ell) + F(b^*) - b^* r_t}{1-v}) + \beta(1-p)u(\frac{s_{b,dd}(\omega|v)(1+r_d-\ell)}{1-v}) >$$
$$u(\omega - s_n(\omega|v)) + p\beta u(s_n(\omega|v)(1-d) + F(b^*) - b^* r_n) + (1-p)\beta u(s_n(\omega|v)(1-d)) =$$
$$u(\omega - s_n(\omega|v')) + p\beta u(s_n(\omega|v')(1-d) + F(b^*) - b^* r_n) + (1-p)\beta u(s_n(\omega|v')(1-d))$$

The first inequality follows from utility maximization given $v'$, the second inequality follows from $U' > 0$, the third inequality follows from $U_b > U_n$ for $v$, and the fourth equality follows from the fact that $v$ does not enter the utility function for the unbanked.

The proof is identical for $U_{b,cd}$.

If $U_{b,dc} > U_n$ for $v$, then $U_{b,dc} > U_n$ if $v' > v$ because $U_{b,dc}$ is constant in $v$. The proof is identical for $U_{b,cc}$.

$r_c$: For fixed $\omega$, we show that if $U_{b,cd} > U_n$ for $r_c$, then $U_{b,cd} > U_n$ if $r_c' > r_c$.

$$
\begin{aligned}
&u(\frac{\omega - s_{b,cd}(\omega|r_c') - C}{1-v}) + p\beta u(\frac{s_{b,cd}(\omega|r_c')(1+r_c') + F(b^*) - b^* r_t}{1-v}) + \beta(1-p)u(\frac{s_{b,cd}(\omega|r_c')(1+r_c')}{1-v}) > \\
&u(\frac{\omega - s_{b,cd}(\omega|r_c) - C}{1-v}) + p\beta u(\frac{s_{b,cd}(\omega|r_c)(1+r_c') + F(b^*) - b^* r_t}{1-v}) + \beta(1-p)u(\frac{s_{b,cd}(\omega|r_c)(1+r_c')}{1-v}) > \\
&u(\frac{\omega - s_{b,cd}(\omega|r_c) - C}{1-v}) + p\beta u(\frac{s_{b,cd}(\omega|r_c)(1+r_c) + F(b^*) - b^* r_t}{1-v}) + \beta(1-p)u(\frac{s_{b,cd}(\omega|r_c)(1+r_c)}{1-v}) > \\
&\qquad u(\omega - s_n(\omega|r_c)) + p\beta u(s_n(\omega|r_c)(1-d) + F(b^*) - b^* r_n) + (1-p)\beta u(s_n(\omega|r_c)(1-d)) = \\
&\qquad\quad u(\omega - s_n(\omega|r_c')) + p\beta u(s_n(\omega|r_c')(1-d) + F(b^*) - b^* r_n) + (1-p)\beta u(s_n(\omega|r_c')(1-d))
\end{aligned}
$$

The first inequality follows from utility maximization given $r_c'$, the second inequality follows from $U' > 0$, the third inequality follows from $U_b > U_n$ for $r_c$, and the fourth equality follows from the fact that $r_c$ does not enter the utility function for the unbanked.

The proof is identical for $U_{b,cc}$.

If $U_{b,dd} > U_n$ for $v$, then $U_{b,dd} > U_n$ if $v' > v$ because $U_{b,dd}$ is constant in $v$. The proof is identical for $U_{b,dc}$.

$r_t$: For fixed $\omega$, we show that if $U_{b,cd} > U_n$ for $r_t$, then $U_{b,cd} > U_n$ if $r_t' > r_t$.

$$
\begin{aligned}
&u(\frac{\omega - s_{b,cd}(\omega|r_t') - C}{1-v}) + p\beta u(\frac{s_{b,cd}(\omega|r_t')(1+r_c) + F(b^*) - b^* r_t'}{1-v}) + \beta(1-p)u(\frac{s_{b,cd}(\omega|r_t')(1+r_c)}{1-v}) > \\
&u(\frac{\omega - s_{b,cd}(\omega|r_t) - C}{1-v}) + p\beta u(\frac{s_{b,cd}(\omega|r_t)(1+r_c) + F(b^*) - b^* r_t'}{1-v}) + \beta(1-p)u(\frac{s_{b,cd}(\omega|r_t)(1+r_c)}{1-v}) > \\
&u(\frac{\omega - s_{b,cd}(\omega|r_t) - C}{1-v}) + p\beta u(\frac{s_{b,cd}(\omega|r_t)(1+r_c) + F(b^*) - b^* r_t}{1-v}) + \beta(1-p)u(\frac{s_{b,cd}(\omega|r_t)(1+r_c)}{1-v}) > \\
&\qquad u(\omega - s_n(\omega|r_t)) + p\beta u(s_n(\omega|r_t)(1-d) + F(b^*) - b^* r_n) + (1-p)\beta u(s_n(\omega|r_t)(1-d)) = \\
&\qquad\quad u(\omega - s_n(\omega|r_t')) + p\beta u(s_n(\omega|r_t')(1-d) + F(b^*) - b^* r_n) + (1-p)\beta u(s_n(\omega|r_t')(1-d))
\end{aligned}
$$

The first inequality follows from utility maximization given $r_t'$, the second inequality follows from $U' > 0$, the third inequality follows from $U_b > U_n$ for $r_t$, and the fourth equality follows from the fact that $r_t$ does not enter the utility function for the unbanked.

The proof is identical for $U_{b,dd}$.

If $U_{b,dc} > U_n$ for $r_t$, then $U_{b,dc} > U_n$ if $r'_t > r_t$ because $U_{b,dc}$ is constant in $r_t$. The proof is identical for $U_{b,cc}$.

The proofs for the remaining variables follow from Proposition A.1. □

## B.13 Proof of Proposition 2.8

From Proposition A.2, we have that household demand for lending, $L(r_d)$, is decreasing in loan interest rates $(r_n, r_g, r_b)$. Loan interest rates are tied to the deposit interest rate $r_d$ by Equations 32, 33, and 34.

From Proposition A.5, we have that the supply of deposits, $D(r_d)$, is increasing in deposit interest rate $r_d$.

Thus, there exists rates $\{r_d^*, r_n^*, r_g^*, r_b^*\}$ such that demand matches supply and markets clear. If $L(r_d)$ and $D(r_d)$ are continuous then $L(r_d^*) = mD(r_d^*)$. Otherwise, $L(r_d^*) \leq mD(r_d^*)$ at equilibrium prices $\{r_d^*, r_n^*, r_g^*, r_b^*\}$ and $L(r_d^* - \epsilon) > mD(r_d * -\epsilon)$ for any $\epsilon > 0$ (i.e. $\{r_d^*, r_n^*, r_g^*, r_b^*\}$ are the lowest prices such that there are enough deposits to meet demand for lending). □

## B.14 Lemma B.2

**Lemma B.2.** *$s_i^c(r)$ is increasing in $\omega(i)$.*

The FOC with respect to savings of those who open a bank account and save in CBDC is given by:

$$(1-v)^{-1}u'(\frac{\omega - s - C}{1-v}) = (1-v)^{-1}(1+r_d-\ell)p\beta u'(\frac{s(1+r_d-\ell)+F(b)-r_nb}{1-v}) \\ +(1-v)^{-1}(1+r_d-\ell)(1-p)\beta u'(\frac{s(1+r_d-\ell)}{1-v})$$

Since the LHS is increasing in $\omega$ and the RHS is constant in $\omega$. $s$ must increase as $\omega$ increases because the LHS is decreasing in $s$.

The proof is identical for $U_{b,cd}$, $U_{b,dc}$, and $U_{b,cc}$. □

## B.15 Proof of Proposition 3.1

Consider an equilibrium where all households have wealth $\omega(i) > \bar{\omega}^0$ and $\Delta L < 0$. Assume $u(x) = log(x)$.

If aggregate welfare increases, then we are done. Otherwise, increase $v$ until aggregate welfare is positive. Household utility under CBDC is increasing in $v$, while utility under the baseline scenario with no CBDC is constant in $v$.

**Lemma B.3.** *$U_i^c$ increases in $v$. $U_i^0$ is constant in $v$*

Let $v' > v$. First, We show that then $U_{b,dd}(v') > U_{b,dd}(v)$.

$$
\begin{aligned}
&U_{b,dd}(v') = \\
&u\left(\frac{\omega - s_{b,dd}(\omega|v') - C}{1-v'}\right) + p\beta u\left(\frac{s_{b,dd}(\omega|v')(1+r_d-\ell) + F(b^*) - b^* r_t}{1-v'}\right) + \beta(1-p)u\left(\frac{s_{b,dd}(\omega|v')(1+r_d-\ell)}{1-v'}\right) \geq \\
&u\left(\frac{\omega - s_{b,dd}(\omega|v) - C}{1-v'}\right) + p\beta u\left(\frac{s_{b,dd}(\omega|v)(1+r_d-\ell) + F(b^*) - b^* r_t}{1-v'}\right) + \beta(1-p)u\left(\frac{s_{b,dd}(\omega|v)(1+r_d-\ell)}{1-v'}\right) > \\
&u\left(\frac{\omega - s_{b,dd}(\omega|v) - C}{1-v}\right) + p\beta u\left(\frac{s_{b,dd}(\omega|v)(1+r_d-\ell) + F(b^*) - b^* r_t}{1-v}\right) + \beta(1-p)u\left(\frac{s_{b,dd}(\omega|v)(1+r_d-\ell)}{1-v}\right) \\
&U_{b,dd}(v)
\end{aligned}
$$

The first inequality follows from utility maximization given $v'$ and the second inequality follows from $v' > v$. Thus, $U_{b,dd}(v') \geq U_{b,dd}(v)$.

The proof is identical for $U_{b,cd}$.

Utility for all other choices under CBDC issuance is constant in $v$. Thus, $U_i^c$ increases in $v$.

$v$ does not enter utility in the baseline scenario without CBDC, thus $U^0$ is constant in $v$. □

Increasing $v$ does not impact the change in lending, $\Delta L$. From Equation 37, we have that there are no possible new deposits from the previously unbanked and outflows of deposits from previously banked households saving in CBDC is unchanged in $v$. Increasing $v$ does not impact the level of savings among the previous banked for $u(x) = log(x)$ as $v$ does not enter the FOC with respect to savings.

The FOC with respect to savings for of those who open a bank account and save in CBDC is given by:

$$(1-v)^{-1}u'(\frac{\omega-s-C}{1-v}) = (1-v)^{-1}(1+r_d-\ell)p\beta u'(\frac{s(1+r_d-\ell)+F(b)-r_nb}{1-v})$$
$$+(1-v)^{-1}(1+r_d-\ell)(1-p)\beta u'(\frac{s(1+r_d-\ell)}{1-v}) =$$
$$\frac{1}{\omega-s-C} = (1+r_d-\ell)p\beta\frac{1}{s(1+r_d-\ell)+F(b)-r_nb}$$
$$+(1+r_d-\ell)(1-p)\beta\frac{1}{s(1+r_d-\ell)})$$

Similar for $U_{b,cd}$, $U_{b,dc}$, and $U_{b,cc}$.

## B.16 Proof of Proposition 4.1

There exists thresholds $\bar{\omega}^{p1,t}$ and $\bar{\omega}^{p2,t}$ following the same proof concept as in Proposition 2.1. It remains to show that:
(1) $\bar{\omega}^{p2,t} \geq \bar{\omega}^{c,t}$ for $t \in \{g, b\}$
(2) $\bar{\omega}^{p1,t} \leq \bar{\omega}^{c,t}$ for $t \in \{g, b\}$
(3) $\bar{\omega}^{p2,t} \lesseqgtr \bar{\omega}^{0}$
(4) $\bar{\omega}^{p2,g} < \bar{\omega}^{p2,b}$

First, consider $\omega > \bar{\omega}^{p2,t}$. This implies that $\max\{U_{b,dd}, U_{b,cd}, U_{b,dc}, U_{b,cc}\} > \max\{U_p, U_n\} \geq U_n$. So, households with $\omega > \bar{\omega}^{p2,t}$ will always open a bank account in the CBDC scenario where only banks distribute CBDC. This implies $\bar{\omega}^{p2,t} \geq \bar{\omega}^{c,t}$, proving (1).

Next, consider $\omega = \bar{\omega}^{c,t}$.

If $U_p > \max\{U_n, U_{b,dd}, U_{b,cd}, U_{b,dc}, U_{b,cc}\}$, then $U_p = \max\{U_n, U_{b,dd}, U_{b,cd}, U_{b,dc}, U_{b,cc}\}$, or the household with $\bar{\omega}^{c,t}$ opens a CBDC wallet with a non-bank PSP. In this case, $\bar{\omega}^{p1,t} < \bar{\omega}^{c,t}$.

If $U_p < \max\{U_n, U_{b,dd}, U_{b,cd}, U_{b,dc}, U_{b,cc}\}$, then $\max\{U_n, U_p, U_{b,dd}, U_{b,cd}, U_{b,dc}, U_{b,cc}\}$ $= \max\{U_n, U_{b,dd}, U_{b,cd}, U_{b,dc}, U_{b,cc}\}$. $\omega = \bar{\omega}^{c,t}$ implies $U_n = \max\{U_{b,dd}, U_{b,cd}, U_{b,dc}, U_{b,cc}\}$. This means that $\max\{U_n, U_{b,dd}, U_{b,cd}, U_{b,dc}, U_{b,cc}\} = \max\{U_{b,dd}, U_{b,cd}, U_{b,dc}, U_{b,cc}\}$. Together, $\max\{U_{b,dd}, U_{b,cd}, U_{b,dc}, U_{b,cc}\} = \max\{U_n, U_p U_{b,dd}, U_{b,cd}, U_{b,dc}, U_{b,cc}\}$ or the household with $\bar{\omega}^{c,t}$ opens a bank account. In this case, $\bar{\omega}^{p2,t} \leq \bar{\omega}^{c,t}$. Since $\bar{\omega}^{p1,t} \leq \bar{\omega}^{p2,t}$, we have that $\bar{\omega}^{p1,t} \leq \bar{\omega}^{c,t}$.

Together, we have that $\bar{\omega}^{p1,t} \leq \bar{\omega}^{c,t}$ in all cases, proving (2).

Next, consider $\omega > \bar{\omega}^{p2,b}$. This implies that $\max\{U_{b,dd}, U_{b,cd}, U_{b,dc}, U_{b,cc}\} > \max\{U_p, U_n\} \geq$

$U_n$ for type $b$ households.

To prove (3), first we show a case where $\bar{\omega}^{p2,t} < \bar{\omega}^0$. Consider $\omega = \bar{\omega}^0$.

Let $v = 0$ and $r_n = r_g = r_b$ (no value from CBDC), and $r_c = r_d - \ell$ (same return from saving in CBDC or deposit account). There is no difference in utility from choosing to save in CBDC vs deposits or spend in CBDC vs cash given having a bank account.

Let $s^p$ be the optimal saving for households with a PSP CBDC wallet and $s^b$ be the optimal saving for households with a bank account.

Utility from opening a bank account under CBDC issuance is

$$u(\omega - s^b - C) + \beta(pu(s^b(1 + r_d - \ell) + \pi(r_n)) + (1 - p)u(s^b(1 + r_d - \ell))$$

Utility from opening a non-bank PSP CBDC account is

$$\begin{aligned} U_p = u(\omega - s^p - C') + \beta(pu(s^p(1 + r_c) + \pi(r_n)) + (1 - p)u(s^p(1 + r_c)) \geq \\ u(\omega - s^b - C') + \beta(pu(s^b(1 + r_c) + \pi(r_n)) + (1 - p)u(s^b(1 + r_c)) > \\ u(\omega - s^b - C) + \beta(pu(s^b(1 + r_d - \ell) + \pi(r_n)) + (1 - p)u(s^b(1 + r_d - \ell)) = U_b \end{aligned}$$

where the first inequality holds from utility maximization and the second inequality follows from $C' < C$ and $r_c = r_d - \ell$

Since $\omega = \bar{\omega}^0$, $U_b = U_n$. Thus, we have $U_p > U_b$. This implies that the household with $\omega = \bar{\omega}^0$ who opens bank account in the baseline scenario without CBDC, opens a non-bank PSP CBDC wallet instead when CBDC is issued with the PSP option. Therefore, $\bar{\omega}^{p2,t} > \bar{\omega}^0$.

Now, we show a case where $\bar{\omega}^{p2,t} > \bar{\omega}^0$. Consider $\omega = \bar{\omega}^{p2,t}$.

Let $r_n = r_g = r_b$ and $r_c = r_d - \ell$ (same return from saving in CBDC or deposit account). There is no difference in utility from choosing to save in CBDC vs deposits or spend in CBDC vs cash given having a bank account. Since $r_n = r_g = r_b$, all households choose to use CBDC for payments.

Utility from opening a bank account under CBDC issuance is

$$\begin{aligned} U_b = u(\omega - s^b - C) + \beta(pu(s^b(1 + r_d - \ell) + \pi(r_n)) + (1 - p)u(s^b(1 + r_d - \ell)) = \\ u(\omega - s^b - C) + \beta(pu(s^b(1 + r_c) + \pi(r_n)) + (1 - p)u(s^p(1 + r_c)) \end{aligned}$$

which is constant in $v$.

Utility from opening a non-bank PSP CBDC account is

$$U_p = u(\frac{\omega - s^p - C'}{1-v}) + \beta(pu(\frac{s^p(1+r_c)+\pi(r_n)}{1-v}) + (1-p)u(\frac{s^p(1+r_c)}{1-v}) >$$
$$u(\frac{\omega - s^b - C'}{1-v}) + \beta(pu(\frac{s^b(1+r_c)+\pi(r_n)}{1-v}) + (1-p)u(\frac{s^b(1+r_c)}{1-v})$$

Note that the second line is increasing in $v$ and the rest of the variables are constants.

Let $v$ be large enough such that

$$u(\frac{\omega - s^b - C'}{1-v}) + \beta(pu(\frac{s^b(1+r_c)+\pi(r_n)}{1-v}) + (1-p)u(\frac{s^b(1+r_c)}{1-v}) > U_b$$

so $U_p > U_b$.

Since $\omega = \bar{\omega}^{p2,t}$ gives us that $U_p = U_n$, we have $U_n > U_b$. This implies that the household with $\omega = \bar{\omega}^{p2,t}$ who opens bank account when CBDC is issued with the PSP option, doesn't in the baseline scenario without CBDC. Therefore, $\bar{\omega}^{p2,t} < \bar{\omega}^0$.

(4) follows from the same proof concept in Proposition A.5. □

## B.17 Proof of Proposition A.6

(1): Assume household is type $g$.

$$U_{p,s} = u(\frac{\omega - s(\omega|r_g) - C'}{1-v}) + p\beta u(\frac{s(\omega|r_g)(1+r_c)+\pi(r_g)}{1-v}) + \beta(1-p)u(\frac{s_{p,s}(\omega|r_g)(1+r_c)}{1-v}) \geq$$
$$u(\frac{\omega - s(\omega|r_n) - C'}{1-v}) + p\beta u(\frac{s(\omega|r_n)(1+r_c)+\pi(r_g)}{1-v}) + \beta(1-p)u(\frac{s(\omega|r_n)(1+r_c)}{1-v}) >$$
$$u(\frac{\omega - s(\omega|r_n) - C'}{1-v}) + p\beta u(\frac{s(\omega|r_n)(1+r_c)+\pi(r_n)}{1-v}) + \beta(1-p)u(\frac{s(\omega|r_n)(1+r_c)}{1-v})$$

where $\pi(r) = \max_b F(b) - br$, note that $\pi(r_n) < \pi(r_g)$ i.f.f. $r_n > r_g$ from Proposition A.2.

The first inequality follows from utility maximization given $r_g$ and the second inequality follows from $r_n > r_g$.

(2): Assume household is type $b$. $U_{p,s} = U_p$ follows from the identical utility maximization problems. □

## B.18 Proof of Proposition 4.2

There exists thresholds $\bar{\omega}^{s1,t}$ and $\bar{\omega}^{s2,t}$ following the same proof concept as in Proposition 2.1. It remains to show that:
(1) $\bar{\omega}^{s1,g} < \bar{\omega}^{p1,g}$ (additional incentive for $g$-types to open PSP CBDC wallet).
(2) $\bar{\omega}^{s1,b} = \bar{\omega}^{p1,b}$ (no additional incentive for $b$-types).

For (1), let $\omega = \bar{\omega}^{p1,g}$, then $U_p = U^p = \max\{U_n, U_p, U_{b,dd}, U_{b,cd}, U_{b,dc}, U_{b,cc}\}$. Proposition 4.2 implies $U_{p,s} > U_p$ for household of type $g$. Thus, $U_{p,s} = \max\{U_n, U_p, U_{p,s}, U_{b,dd}, U_{b,cd}, U_{b,dc}, U_{b,cc}\}$ so the $g$ type household with $\omega = \bar{\omega}^{p1,g}$ always opens a PSP CBDC wallet when data sharing is allowed. Thus, $\bar{\omega}^{s1,g} < \bar{\omega}^{p1,g}$.

(2) follows from identical utility maximization problems. □

# C Appendix Tables

**Table A1:** Baseline Parameters

| | |
|---|---|
| $p_g$ | 0.95 |
| $p_b$ | 0.55 |
| $C$ | 0.2 |
| $r_c$ | 0 |
| $\beta$ | 0.9 |
| $d$ | 0.03 |
| $v$ | 0.003 |
| $m$ | 1 |
| $\phi$ | 0.5 |
| $C'$ | 0.175 |
| $\alpha$ | 3 |
| $L$ | 10 |
| $H$ | 50 |
| $\lambda$ | 50 |
| $q$ | 0.5 |
| $L'$ | 0.17 |
| $H'$ | 0.34 |
| $r_w$ | 0.02 |
| $\delta$ | 0.01 |
| $\Omega$[1] | 40 |
| $W$[1] | 10000 |

*Notes: This table presents the baseline parameters for the calibration exercise.* [1] *For robustness. Similar results for all* $\Omega \in [40, H]$ *and* $W \in [0, 10000]$.

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